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Electroweak Interaction with Singlet Quarks

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Contents

1 Introduction **1**

2 Electroweak interaction **5**

2.1 The standard model 5

2.2 Masses and mixings of quarks and leptons 7

2.3 Flavor changing interactions of quarks 7

2.3.1 Charged current interaction 8

2.3.2 Neutral current interaction 8

2.4 Vector-like fermions 9

3 Quark mixings and flavor changing interactions with singlet quarks **11**

3.1 Introduction 11

3.2 Electroweak model with singlet quarks 13

3.3 Quark masses and mixings with singlet quarks 14

3.3.1 Choices of the quark basis 15

3.3.2 Basis (a) with $\Delta'_{qQ} = \mathbf{0}$ 17

3.3.3 Basis (b) with $\Delta_{qQ} = \mathbf{0}$ 18

3.3.4 Seesaw model 19

3.3.5 Relations among the three bases 22

3.4 Flavor changing interactions 25

3.4.1 Charged currents 26

3.4.2 Neutral currents 27

3.5 Numerical analysis 35

3.6 Summary and discussion 44

Appendix A Diagonalization of the quark mass matrix 46

4 Universality of strength for Yukawa couplings with extra down-type quark singlets **56**
4.1 Introduction 56
4.2 Quark masses and mixings with extra down-type quarks in USY 57
4.3 Numerical result for the CKM matrix 62
4.4 Discussion and conclusion 64

5 Flavor-changing interactions with singlet quarks and their implications for the LHC **66**
5.1 Introduction 66
5.2 Quark mixings and flavor-changing interactions 68
5.2.1 Quark masses and mixings 69
5.2.2 Flavor-changing interactions 71
5.3 Singlet quark Effects in $\Delta F = 2$ mixings of neutral mesons 74
5.4 Decays of singlet quarks and Higgs particles 84
5.4.1 Singlet quark decays 85
5.4.2 Higgs particle decays 94
5.5 Summary 96
Appendix B Relations among the gauge and scalar couplings 96

6 Summary and conclusions **100**

Acknowledgments **102**

References **103**

Chapter 1

Introduction

So far the standard model has been established in explaining almost all the elementary particle phenomena, which is based on the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge theories (see for a review [1]). The $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge theory provides the dynamics of the electromagnetic and weak interactions (electroweak interaction, synthetically) through the spontaneous symmetry breaking induced by the Higgs field, while the $\mathrm{SU}(3)_\mathrm{C}$ gauge theory describes the quantum chromodynamics (QCD) for the strong interaction. These interactions are mediated by the vector gauge fields or gauge bosons. On the other hand, the fundamental matter fields or particles include quarks and leptons (fermions) with spin 1/2, and also the Higgs bosons (scalars) with spin 0. Specifically, these matter fields are described as follows in terms of the representations (quantum numbers) of the gauge groups $(\mathrm{SU}(3)_C, \mathrm{SU}(2)_W, \mathrm{U}(1)_Y)$:

$$
\begin{aligned}
&\text{Quarks}: \ q=(u,d)\ (\mathbf{3},\mathbf{2},1/6),\ u^c\ (\mathbf{3}^*,\mathbf{1},-2/3),\ d^c\ (\mathbf{3}^*,\mathbf{1},1/3),\\
&\text{Leptons}: \ l=(\nu,e)\ (\mathbf{1},\mathbf{2},-1/2),\ e^c\ (\mathbf{1},\mathbf{1},1),\\
&\text{Higgs bosons}: \ H=(H^+,H^0)\ (\mathbf{1},\mathbf{2},1/2),
\end{aligned}
$$

where the quarks and leptons are presented with left-handed Weyl fields, and the superscript "c" indicates the anti-fermion or the charge conjugate of the right-handed fermion. It is known that there are three generations of quarks and leptons, namely $(u_1,u_2,u_3) = (u,c,t)$, $(d_1,d_2,d_3) = (d,s,b)$, $(e_1,e_2,e_3) = (e,\mu,\tau)$, $(\nu_1,\nu_2,\nu_3) = (\nu_e,\nu_\mu,\nu_\tau)$. These quarks and leptons are distinguished with suitable quantum numbers so-called *flavors*.

The quarks and leptons, except for the neutrinos, acquire masses by coupling with the Higgs bosons (Yukawa couplings) through the spontaneous breaking of the $\mathrm{SU}(2)_\mathrm{W} \times \mathrm{U}(1)_\mathrm{Y}$

symmetry. Their masses are currently determined by experiments as follows [1]:

$$
\begin{aligned}
&m_u = 1.7 - 3.1\text{MeV},\ m_c = 1.18 - 1.34\text{GeV},\ m_t = 171.4 - 174.4\text{GeV},\\
&m_d = 4.1 - 5.7\text{MeV},\ m_s = 80 - 130\text{MeV},\ m_b = 4.13 - 4.37\text{GeV},\\
&m_e = 0.5110\text{MeV},\ m_\mu = 105.66\text{MeV},\ m_\tau = 1.777\text{GeV},\\
&m_{\nu_e} < 2\text{eV},\ m_{\nu_\mu} < 0.19\text{MeV},\ m_{\nu_\tau} < 18.2\text{MeV}.
\end{aligned}
$$

We clearly find certain hierarchies among the quark and lepton masses,

$$
\begin{aligned}
&\bullet\ m_u \ll m_c \ll m_t,\ \bullet\ m_d \ll m_s \ll m_b,\ \bullet\ m_s \ll m_c,\ \bullet\ m_b \ll m_t,\\
&\bullet\ m_e \ll m_\mu \ll m_\tau,\ \bullet\ m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau} \simeq 0 \ll m_e.
\end{aligned}
$$

Here, it should be remarked that the existence of tiny but nonzero neutrino masses has been confirmed recently by the neutrino oscillation experiments.

Through the mass generation process, the mixing of quarks with the same charge also occur among the generations. Then, this mixing causes the flavor changing processes in the weak interaction. In the standard model, they appear only in the coupling of the quarks with the charged weak bosons $W^\pm$ at the tree level, which is described with the Cabibbo-Kobayashi-Maskawa (CKM) matrix (a 3×3 unitary matrix) for the transition between (u, c, t) and (d, s, b). (See for a review [1].) Such flavor changing interactions contribute to the β decays. Meanwhile, provided the nonzero neutrino masses have been confirmed by the neutrino oscillation experiments, the lepton mixing is described in a similar fashion to the quark mixing.

Despite its successful features, the standard model still involve some issues to be clarified. For instance, some elements of the CKM matrix involving the third generation have not been determined yet experimentally. As the essential ingredient for the $\mathrm{SU(2)_W} \times \mathrm{U(1)_Y}$ symmetry breaking, the Higgs particle should be discovered, probably by the Large Hadron Collider (LHC) at CERN. There is no guiding principle for the mass hierarchies and mixings of the quarks and leptons. The neutrino masses cannot be generated within the standard model.

According to these unsolved features of the standard model, various extensions have been considered so far. In the grand unified theory, the $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge groups are embedded into a simple gauge group such as SU(5), SO(10) or E_6 (see for a review [1]). Specifically in the E_6 model [2, 3, 4], the quarks, leptons, and Higgs bosons are described in the **27** dimensional representation. Then, it is noteworthy that some exotic particles such as the $\mathrm{SU}(2)_W$ singlet quarks are also included in the **27**. The presence of these particles is also suggested by the extra-dimension theories and the superstring theories. The extension of electroweak interaction with singlet quarks provides new physics, which does not exist in the standard model, in particular, violation of the unitarity of the CKM matrix, and the flavor changing neutral currents (FCNC's) at the tree level [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

In this thesis, we investigate a variety of physical implications which are provided by the extension of the standard model including the singlet quarks. They are expected to be revealed as new physics in the future experiments. Especially, the discovery of singlet quarks is directly expected at the LHC, which has started operation reaching the energy 14 TeV. This thesis is based on the studies in Refs. [26], [27], [28]. The remaining parts of the thesis are presented as follows.

In Chapter 2, we review some topics about the electroweak interaction, including the singlet quarks.

In Chapter 3, we investigate the aspects of the quark mixings and flavor changing interactions in electroweak models with singlet quarks. The effects on the ordinary quark mixing are determined in terms of the quark masses and the parameters describing the mixing between the ordinary quarks q and the singlet quarks Q (q-Q mixing). Some salient features arise in the flavor changing interactions through the q-Q mixing. The unitarity of the CKM matrix within the ordinary quark sector is violated, and the FCNC's appear both in the gauge and scalar couplings. The flavor changing interactions are calculated appropriately in terms of the q-Q mixing parameters and the quark masses, which really exhibit specific flavor structures. It is found that there are reasonable ranges of the model parameters to reproduce the ordinary quark mass hierarchy and the actual CKM structure even in the presence of q-Q mixing. Some phenomenological effects of the singlet quarks are also discussed. In particular, the scalar FCNC's may be more important in some cases, if the singlet quarks as well as the extra scalar particles from the singlet Higgs fields have masses $\sim 100\mathrm{GeV} - 1\mathrm{TeV}$.

In Chapter 4, we investigate the quark masses and mixings by including the down-type quark singlets in universality of strength for Yukawa couplings (USY). In contrast with the standard model with USY, the sufficient CP violation is obtained for the CKM matrix through the mixing between the ordinary quarks and quark singlets. The top-bottom mass hierarchy $m_t \gg m_b$ also appears naturally in the USY scheme with the down-type quark singlets.

In Chapter 5, we investigate the flavor changing interactions in the extension of the standard model with singlet quarks and singlet Higgs, which are induced by the q-Q mixing. We consider the effects of the gauge and scalar interactions in the $\Delta F = 2$ mixings of K^0, B_d, B_s and D^0 mesons to show the currently allowed range of the q-Q mixing. Then, we investigate the new physics around the electroweak scale to the TeV scale, which is accessible to the LHC. Especially, the scalar coupling mediated by the singlet Higgs may provide distinct signatures for the decays of the singlet quarks and Higgs particles, which should be compared with the conventionally expected ones via the gauge and standard Higgs couplings. Observations of the singlet quarks and Higgs particles will present us important insights on the q-Q mixing and Higgs mixing.

Chapter 6 is devoted to summary and conclusions.

Chapter 2

Electroweak interaction

2.1 The standard model

The standard model describes the physics of elementary particles based on the gauge symmetries $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, namely the $\mathrm{SU}(3)_C$ for the strong interaction and the $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ for the electroweak interaction. Henceforth in this thesis, we concentrate on the physics concerning the electroweak interaction.

The $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ symmetry is spontaneously broken to the electromagnetic $\mathrm{U}(1)_{\mathrm{em}}$ symmetry by the Higgs mechanism. Then, the weak gauge bosons $W^\pm$ and Z acquire masses, while the photon A remains massless. They are described in terms of the original $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge fields W_a ($a = 1, 2, 3$) and B as

$$W^\pm = (W_1 \mp iW_2)/\sqrt{2}, \tag{2.1}$$

$$Z = \cos\theta_W W_3 - \sin\theta_W B, \tag{2.2}$$

$$A = \sin\theta_W W_3 + \cos\theta_W B, \tag{2.3}$$

where $\theta_W = \tan^{-1}(g'/g)$ is the weak angle with the gauge couplings g and g' for $\mathrm{SU}(2)_W$ and $\mathrm{U}(1)_Y$, respectively. The electric charge, the generator of $\mathrm{U}(1)_{\mathrm{em}}$, is given by

$$Q_{\mathrm{em}} = I_3 + Y/2, \tag{2.4}$$

where I_3 is the diagonal generator of the weak isospin $\mathrm{SU}(2)_W$, and Y is the generator of $\mathrm{U}(1)_Y$ as the weak hypercharge.

A variety of particles (fields) are contained in the standard model, specifically the vector bosons with spin 1 mediating the gauge interactions, the quarks and leptons (fermions) with spin 1/2, and the Higgs bosons with spin 0. The fermions and bosons are listed in

Fermions	Electric charge	First generation	Second generation	Third generation
Quarks	$+2/3$	Up (u)	Charm (c)	Top (t)
	$-1/3$	Down (d)	Strange (s)	Bottom (b)
Leptons	-1	Electron (e)	Muon (μ)	Tau (τ)
	0	Electron neutrino (ν_e)	Mu neutrino (ν_μ)	Tau neutrino (ν_τ)

Table 2.1: Fermions. There are three generations of quarks and leptons.

Bosons	Charged bosons	Neutral bosons
Gauge bosons	Charged weak bosons ($W^\pm$)	Neutral weak boson (Z) Photon (A) Gluons (g_α; $\alpha = 1-8$)
Higgs bosons	Charged Higgs (H^+)	Neutral Higgs (H^0)

Table 2.2: Bosons. Here, the charged Higgs bosons $H^\pm$ [$H^- \equiv (H^+)^*$] are absorbed as the longitudinal modes of the massive weak bosons $W^\pm$ through the Higgs mechanism.

Particles		$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_W$	$\mathrm{U}(1)_Y$
Quarks	$q_\mathrm{L} = \begin{pmatrix} u_\mathrm{L} \\ d_\mathrm{L} \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$
	u_R	$\mathbf{3}$	$\mathbf{1}$	$2/3$
	d_R	$\mathbf{3}$	$\mathbf{1}$	$-1/3$
Leptons	$l_\mathrm{L} = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
	e_R	$\mathbf{1}$	$\mathbf{1}$	-1
Higgs bosons	$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$
Gauge bosons	(W_1, W_2, W_3)	$\mathbf{1}$	$\mathbf{3}$	0
	B	$\mathbf{1}$	$\mathbf{1}$	0
	$(g_\alpha;\ \alpha = 1-8)$	$\mathbf{8}$	$\mathbf{1}$	0

Table 2.3: The representations (quantum numbers) of the respective particles. The subscripts “L” and “R” represent the left-handed and right-handed fermions, respectively.

Tables 2.1 and 2.2, respectively. The representations (quantum numbers) of the respective particles are also shown in Table 2.3.

2.2 Masses and mixings of quarks and leptons

The Yukawa couplings of the quarks and leptons with the Higgs double are given by

$$\mathcal{L}_{\text{Yukawa}} = \overline{q^0_{\mathrm{L}}}\lambda_u(i\sigma_2 H^*)u^0_{\mathrm{R}} + \overline{q^0_{\mathrm{L}}}\lambda_d H d^0_{\mathrm{R}} + \overline{l^0_{\mathrm{L}}}\lambda_e H e^0_{\mathrm{R}} + \text{H.c.}, \tag{2.5}$$

where the generation indices are omitted, and the superscript “0” indicates the electroweak interaction basis, which may be different from the mass eigenstate basis without “0”, as seen below. Through the spontaneous breaking of $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ with the vacuum expectation value of the neutral Higgs field $v = \langle H^0 \rangle$, the mass terms of the quarks and leptons are generated as 3×3 matrices,

$$M_u = \lambda_u v,\ M_d = \lambda_d v,\ M_e = \lambda_e v. \tag{2.6}$$

Here, it should be noted that the neutrinos remain massless, lacking the right-handed components in the standard model to form the mass matrix M_ν. The quark mass matrix M_q $(q = u, d)$ is diagonalized by unitarity transformations $V_{q_{\mathrm{L}}}$ and $V_{q_{\mathrm{R}}}$ as

$$\bar{M}_q = V^\dagger_{q_{\mathrm{L}}} M_q V_{q_{\mathrm{R}}} = \begin{pmatrix} m_{q_1} & 0 & 0 \\ 0 & m_{q_2} & 0 \\ 0 & 0 & m_{q_3} \end{pmatrix}, \tag{2.7}$$

where $q_i = u, c, t$ or d, s, b with the generation indices $i = 1, 2, 3$. The mass eigenstates are given by

$$q_{\mathrm{L}} = V^\dagger_{q_{\mathrm{L}}} q^0_{\mathrm{L}},\ q_{\mathrm{R}} = V^\dagger_{q_{\mathrm{R}}} q^0_{\mathrm{R}}. \tag{2.8}$$

The mass matrix of the charged leptons M_e is diagonalized in a similar fashion.

2.3 Flavor changing interactions of quarks

The electroweak interaction of the quarks are described in terms the mass eignenstates. Then, flavor changing couplings appear generally through the quark mixing. (Henceforth, we concentrate on the quarks, while the leptons are treated similarly.)

2.3.1 Charged current interaction

The charged current interaction of the quarks coupling with the weak bosons $W^{\pm}$, at the beginning, has a flavor-diagonal form as $\overline{u_{\rm L}^0}\mathbf{1}d_{\rm L}^0$ in the electroweak interaction basis. Then, it is described in terms of the quark mass eigenstates as

$$\mathcal{L}_{\rm CC}(W) \;=\; \frac{g}{\sqrt{2}}\overline{u_{\rm L}}\gamma^{\mu}V_{\rm CKM}d_{\rm L}W_{\mu}^{+} + {\rm H.c.}, \tag{2.9}$$

involving the CKM matrix $V_{\rm CKM}$. The CKM mixing appears as the difference between the up-type and down-type mixings, which is given by a 3×3 unitary matrix

$$V_{\rm CKM} = V_{u_{\rm L}}^{\dagger}V_{d_{\rm L}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2.10}$$

The CKM matrix represents the flavor changing transitions, namely V_{ud} for $d \to u$ emitting W^-, and so on. It can be parameterized with three mixing angles θ_{12}, θ_{13}, θ_{23} and a CP-violating complex phase δ as

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.11}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$. The magnitude of $V_{\rm CKM}$ is determined experimentally with help of the unitarity condition as (central values)

$$|V_{\rm CKM}| = \begin{vmatrix} 0.9743 & 0.2253 & 0.0035 \\ 0.2252 & 0.9735 & 0.0410 \\ 0.0086 & 0.0403 & 0.9992 \end{vmatrix}. \tag{2.12}$$

Up to now, the matrix elements of $V_{\rm CKM}$ within the first or second generations are determined in good precision. On the other hand, those involving the third generation are determined only with large experimental and theoretical uncertainties.

2.3.2 Neutral current interaction

The neutral current interaction of the quarks coupling with the weak boson Z is given by

$$\begin{aligned}\mathcal{L}_{\rm NC}(Z) \;=\; & \frac{g}{\cos\theta_W}[I_3(q) - \sin^2\theta_W Q_{\rm em}(q)]\overline{q_{\rm L}}\gamma^{\mu}q_{\rm L}Z_{\mu} \\ & -\frac{g\sin^2\theta_W}{\cos\theta_W}Q_{\rm em}(q)\overline{q_{\rm R}}\gamma^{\mu}q_{\rm R}Z_{\mu},\end{aligned} \tag{2.13}$$

where $I_3(u) = 1/2$ and $I_3(d) = -1/2$ for the left-handed components. Note here that in the standard model the quarks with the same electric charge couple to Z with the same amplitude. This implies that the above neutral gauge interaction remains flavor-diagonal even in terms of the quark mass eigenstates, namely no FCNC's appear at the tree level, which is due to the cancellation of the quark mixings as $\overline{q^0_{\mathrm{L,R}}}\mathbf{1}q^0_{\mathrm{L,R}} = \overline{q_{\mathrm{L,R}}}\mathbf{1}q_{\mathrm{L,R}}$ with $V^\dagger_{q_{\mathrm{L,R}}}V_{q_{\mathrm{L,R}}} = \mathbf{1}$ $(q = u, d)$. The flavor changing neutral processes are substantially suppressed, occurring only in loop diagrams as quantum corrections. Hence, the observation of sizable FCNC's will serve as a good evidence of new physics, which may be provided by exotic particles such as the singlet quarks.

2.4 Vector-like fermions

Extensions of the standard model are motivated in various points of view toward the discovery of new physics, including exotic fermions, extra Higgs fields, enlargement of gauge interactions, supersymmetry, superstring, extra dimensions, and so on. Among these intriguing possibilities, the grand unified theories such as the E_6 model predict certain exotic fermions. Specifically, the $\mathbf{27}$ representation of E_6 is decomposed under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y \subset \mathrm{E}_6$ as

$$\begin{aligned}
\mathbf{27} \;=\; & q\ (\mathbf{3},\mathbf{2},1/6) + u^c\ (\mathbf{3}^*,\mathbf{1},-2/3) + d^c\ (\mathbf{3}^*,\mathbf{1},1/3) \\
& + l\ (\mathbf{1},\mathbf{2},-1/2) + e^c\ (\mathbf{1},\mathbf{1},1) \\
& + D\ (\mathbf{3},\mathbf{1},-1/3) + D^c\ (\mathbf{3}^*,\mathbf{1},1/3) \\
& + L\ (\mathbf{1},\mathbf{2},-1/2) + L^c\ (\mathbf{1},\mathbf{2},1/2) \\
& + \nu^c\ (\mathbf{1},\mathbf{1},0) + S\ (\mathbf{1},\mathbf{1},0),
\end{aligned} \tag{2.14}$$

where the fermions are presented with left-handed Weyl fields. Furthermore, the superstring models suggest that other exotic fermions, e.g., U $(\mathbf{3},\mathbf{1},2/3) + U^c$ $(\mathbf{3}^*,\mathbf{1},-2/3)$ may appear from the combination of $\mathbf{27} + \overline{\mathbf{27}}$. These exotic fermions such as the quark singlet D and the lepton doublet L in the above decomposition have the vector couplings to the gauge bosons in the standard model (without axial-vector couplings). This is due to the fact that the left-handed and right-handed (or charge conjugate with superscript "c") components transform in the same way under the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, in contrast with the ordinary quarks and leptons. Hence, these exotic fermions are called *vector-like*. We also note that the $\mathbf{27}$ of E_6 contains two extra fields, ν^c and S, which do not have the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge interaction. Especially, ν^c can be identified with the right-handed neutrino for the neutrino mass generation.

These vector-like quarks and leptons are expected to provide various new physics beyond the standard model, including the unitarity violation of the CKM matrix, the FCNC's, and so on. This motivates the present thesis to study the new physics provided by the singlet quarks in the electroweak interaction.

Chapter 3

Quark mixings and flavor changing interactions with singlet quarks

In this chapter, we investigate the aspects of the quark mixings and flavor changing interactions in electroweak models with singlet quarks. The effects on the ordinary quark mixing are determined in terms of the quark masses and the parameters describing the mixing between the ordinary quarks q and the singlet quarks Q (q-Q mixing). Some salient features arise in the flavor changing interactions through the q-Q mixing. The unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix within the ordinary quark sector is violated, and the flavor changing neutral currents (FCNC's) appear both in the gauge and scalar couplings. The flavor changing interactions are calculated appropriately in terms of the q-Q mixing parameters and the quark masses, which really exhibit specific flavor structures. It is found that there are reasonable ranges of the model parameters to reproduce the ordinary quark mass hierarchy and the actual CKM structure even in the presence of q-Q mixing. Some phenomenological effects of the singlet quarks are also discussed. In particular, the scalar FCNC's may be more important in some cases, if the singlet quarks as well as the extra scalar particles from the singlet Higgs fields have masses $\sim 100\mathrm{GeV} - 1\mathrm{TeV}$.

3.1 Introduction

Extensions of the standard model may be motivated in various points of view toward the discovery of new physics. Among many intriguing possibilities, the presence of isosinglet quarks is suggested in certain models such as E_6 type unified models [2, 3, 4]. Specifically, there are two types of singlet quarks, U with electric charge $Q_{\mathrm{em}} = 2/3$ and D with

$Q_{\rm em} = -1/3$, which may mix with the ordinary quarks. Then, various novel features arise through the mixing between the ordinary quarks ($q = u, d$) and the singlet quarks ($Q = U, D$). The unitarity of Cabibbo-Kobayashi-Maskawa (CKM) matrix within the ordinary quark sector is violated, and the flavor changing neutral currents (FCNC's) appear at the tree-level [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. These flavor changing interactions are actually described in terms of the q-Q mixing parameters and the quark masses, as seen in detail in the following sections. This may be viewed as an interesting extension of the natural flavor conservation proposed in the early literature [29, 30]. Furthermore, the q-Q mixing may involve CP violating phases. Hence, it is quite expected that the q-Q mixing provide significant effects on various physical processes. It is also noted that the so-called seesaw mechanism even works for generating the ordinary quark masses through the q-Q mixing [31, 32, 33, 34, 35, 36, 37, 38, 39].

The q-Q mixing effects on the Z boson mediated neutral currents have been investigated so far extensively in the literature [9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. These analyses show, in particular, that there is a good chance to find the singlet quark effects in the B physics. Some contributions of the neutral couplings mediated by the Higgs scalar particles have also been considered on the neutron electric dipole moment and the neutral meson mixings [10, 24, 25, 38].

The singlet quarks may even provide important contributions in cosmology. In fact, for the electroweak baryogenesis [40] the CP violating q-Q mixing through the coupling with a complex singlet Higgs field S can be efficient to generate the chiral charge fluxes through the bubble wall [41, 42, 43]. This possibility is encouraging, since the CP asymmetry induced by the conventional CKM phase is far too small to account for the observed baryon to entropy ratio. Furthermore, the singlet Higgs field S providing the singlet quark mass term and the q-Q mixing term is preferable for realizing a strong enough first order electroweak phase transition [43].

As mentioned so far, the singlet quarks may bring various intriguing features in particle physics and cosmology. Then, it is worth understanding in detail the characteristic properties of the electroweak models incorporating the singlet quarks. Specifically, it is important to show how the ordinary quark masses and mixings are affected by the q-Q mixing. The structures of the CKM mixing and FCNC's should also be clarified properly. In this chapter, we present systematic and comprehensive descriptions of the quark masses, mixings and flavor changing interactions in the presence of singlet quarks. In Sec.

3.2, a representative model with singlet quarks is presented. In Sec. 3.3, the quark masses and mixings, which are affected by the q-Q mixing, are calculated in detail. Then, the q-Q mixing effects on the gauge and scalar interactions are examined in Sec. 3.4. They are described appropriately in terms of the q-Q mixing parameters and the quark masses. In Sec. 3.5, numerical calculations are performed to confirm the flavor structures of the q-Q mixing effects. Sec. 3.6 is devoted to summary and some discussion on the phenomenological effects provided by the singlet quarks. The technical details in diagonalizing the quark mass matrix are presented in Appendix A.

3.2 Electroweak model with singlet quarks

We first describe a representative electroweak model based on the gauge symmetry $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, where both types of singlet quarks U and D and also one complex singlet Higgs field S are incorporated. The generic Yukawa couplings of quarks are given by

$$\begin{aligned}\mathcal{L}_{\mathrm{Y}} = & - u_0^c \lambda_u \Psi_{q_0} \Phi_H - U_0^c h_u \Psi_{q_0} \Phi_H \\ & - u_0^c (f_U S + f_U' S^\dagger) U_0 - U_0^c (\lambda_U S + \lambda_U' S^\dagger) U_0 \\ & - d_0^c \lambda_d V_0^\dagger \Psi_{q_0} \tilde{\Phi}_H - D_0^c h_d V_0^\dagger \Psi_{q_0} \tilde{\Phi}_H \\ & - d_0^c (f_D S + f_D' S^\dagger) D_0 - D_0^c (\lambda_D S + \lambda_D' S^\dagger) D_0 \\ & + \mathrm{H.c.}\end{aligned} \tag{3.1}$$

in terms of the two-component Weyl fields for the weak eigenstates with subscript "0". (The generation indices and the Lorentz factors are omitted here for simplicity.) The isodoublets of left-handed ordinary quarks are represented by

$$\Psi_{q_0} = \begin{pmatrix} u_0 \\ V_0 d_0 \end{pmatrix} \tag{3.2}$$

with a certain 3×3 unitary matrix V_0, and $V_0^\dagger \Psi_{q_0} \equiv (V_0^\dagger u_0, d_0)^{\mathrm{T}}$. The Higgs doublet is also given by

$$\Phi_H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{3.3}$$

with $\tilde{\Phi}_H \equiv i\tau_2 \Phi_H^*$.

The Higgs fields develop vacuum expectation values (VEV's),

$$\langle H^0 \rangle = \frac{v}{\sqrt{2}}, \ \langle S \rangle = \frac{v_S}{\sqrt{2}} \, \mathrm{e}^{i\phi_S}. \tag{3.4}$$

Here a complex phase ϕ_S is included in $\langle S \rangle$, which may be induced by the CP violation in the Higgs sector either spontaneous or explicit. The quark mass matrix is then produced with these VEV's as

$$\mathcal{M}_{\mathcal{Q}} = \begin{pmatrix} M_q & \Delta_{qQ} \\ \Delta'_{qQ} & M_Q \end{pmatrix}, \tag{3.5}$$

where the submatrices are given by

$$M_q = \frac{1}{\sqrt{2}} \lambda_q v, \tag{3.6}$$

$$\Delta_{qQ} = \frac{1}{\sqrt{2}} (f_Q \mathrm{e}^{i\phi_S} + f'_Q \mathrm{e}^{-i\phi_S}) v_S, \tag{3.7}$$

$$\Delta'_{qQ} = \frac{1}{\sqrt{2}} h_q v, \tag{3.8}$$

$$M_Q = \frac{1}{\sqrt{2}} (\lambda_Q \mathrm{e}^{i\phi_S} + \lambda'_Q \mathrm{e}^{-i\phi_S}) v_S. \tag{3.9}$$

Hereafter, the quarks with the same electric charge are collectively denoted by $\mathcal{Q} = (q, Q)$, i.e., $\mathcal{U} = (u, U)$ and $\mathcal{D} = (d, D)$. The dimensions of $\mathcal{M}_{\mathcal{Q}}$ and its submatrices are specified with the three generations of ordinary quarks and the number N_Q of singlet quarks.

Some remarks may be presented concerning possible variants of the model. It is straightforward to describe the models admitting only either U or D quarks. The complex singlet Higgs field S is employed in the present model to generate the singlet quark mass term and q-Q mixing term. This choice will be motivated, in particular, for the electroweak baryogenesis. The CP violating q-Q mixing provided by the space-dependent complex Higgs field S can be efficient for generating the chiral charge fluxes through the bubble wall [41, 42, 43]. Alternative options, however, may be considered about the singlet Higgs S, including the following cases. (i) The singlet Higgs S is a real scalar field rather than complex one. (ii) The singlet Higgs S is eliminated (or it has a mass much larger than the electroweak scale) while leaving its contributions to the quark mass matrix. (iii) The models with the complex Higgs S may be extended by incorporating supersymmetry. The following investigations are made in detail for the model given in Eq. (3.1) with one complex S. The results can be extended readily for these variants of the model, as will be mentioned occasionally.

3.3 Quark masses and mixings with singlet quarks

We present in this section the detailed description of the quark masses and mixings which are affected by the q-Q mixing. It is desired that even in the presence of singlet quarks the ordinary quark mass hierarchy and the CKM structure are reproduced in some reasonable regions of the model parameter space. This issue will be addressed in the following by inspecting systematically the form of quark mass matrix $\mathcal{M}_{\mathcal{Q}}$ and its diagonalization.

The total quark mass matrix is diagonalized as usual by unitary transformations $\mathcal{V}_{\mathcal{Q}_{\mathrm{L}}}$ and $\mathcal{V}_{\mathcal{Q}_{\mathrm{R}}}$:

$$\mathcal{V}^{\dagger}_{\mathcal{Q}_{\mathrm{R}}}\mathcal{M}_{\mathcal{Q}}\mathcal{V}_{\mathcal{Q}_{\mathrm{L}}} = \begin{pmatrix} \bar{M}_q & \mathbf{0} \\ \mathbf{0} & \bar{M}_Q \end{pmatrix}, \tag{3.10}$$

where

$$\bar{M}_q = \mathrm{diag.}(m_{q_1}, m_{q_2}, m_{q_3}), \tag{3.11}$$

$$\bar{M}_Q = \mathrm{diag.}(m_{Q_1}, \cdots), \tag{3.12}$$

and $(q_1, q_2, q_3) = (u, c, t)$ or (d, s, b). The quark mass eigenstates are given by

$$\begin{pmatrix} q \\ Q \end{pmatrix} = \mathcal{V}^{\dagger}_{\mathcal{Q}_{\mathrm{L}}} \begin{pmatrix} q_0 \\ Q_0 \end{pmatrix}, \tag{3.13}$$

$$(q^c, Q^c) = (q^c_0, Q^c_0)\mathcal{V}_{\mathcal{Q}_{\mathrm{R}}}. \tag{3.14}$$

The relevant $(3 + N_Q) \times (3 + N_Q)$ unitary matrices are given by

$$\mathcal{V}_{\mathcal{Q}_\chi} = \begin{pmatrix} V_{q_\chi} & \epsilon_{q_\chi} \\ -\epsilon'^{\dagger}_{q_\chi} & V_{Q_\chi} \end{pmatrix} \tag{3.15}$$

for the respective chirality sectors $\chi = \mathrm{L}, \mathrm{R}$. The $3 \times N_Q$ matrices ϵ_{q_χ} and ϵ'_{q_χ} represent the q-Q mixing.

The leading order calculations on the quark masses and mixings with singlet quarks are given for $N_Q = 1$ in the literature [5, 6, 7, 8]. Here, we would rather like to present comprehensive understandings on the q-Q mixing effects which are even valid beyond the leading orders for more general cases including several singlet quarks ($N_Q \geq 1$).

3.3.1 Choices of the quark basis

We first note that the quark mass matrix $\mathcal{M}_{\mathcal{Q}}$ may be reduced to a specific form with either $\Delta_{qQ} = \mathbf{0}$ or $\Delta'_{qQ} = \mathbf{0}$ by a unitary transformation of the right-handed quarks, which are undistinguishable by means of the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$. Then, the Yukawa coupling λ_q can be made diagonal and non-negative by unitary transformations of the ordinary quarks:

$$\lambda_q = \mathrm{diag.}(\lambda_{q_1}, \lambda_{q_2}, \lambda_{q_3}). \tag{3.16}$$

The other couplings f_Q, f'_Q and h_q involving the ordinary quarks as well as the mixing matrix V_0 are redefined accordingly. The condition $\Delta_{qQ} = \mathbf{0}$ or $\Delta'_{qQ} = \mathbf{0}$ is, however, maintained by these transformations of the ordinary quarks. In this basis, by turning off the q-Q mixing with $f_Q, f'_Q, h_q \to 0$, the quark fields u_0 and d_0 are reduced to the mass eigenstates, and V_0 is identified with the CKM matrix. The actual CKM matrix V is slightly modified from V_0 by the q-Q mixing, as shown explicitly later.

It should be noticed that the quark transformations made so far to specify the form of $\mathcal{M}_\mathcal{Q}$ do not mix the electroweak doublets with the singlets, respecting the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge symmetry. Hence, without loss of generality we may have two appropriate choices of the quark basis for the electroweak eigenstates:

$$\textbf{basis (a)} \quad : \quad \Delta'_{qQ} = \mathbf{0}, \tag{3.17}$$

$$\textbf{basis (b)} \quad : \quad \Delta_{qQ} = \mathbf{0}. \tag{3.18}$$

In either case, with the diagonal λ_q coupling we have the submatrix of $\mathcal{M}_\mathcal{Q}$,

$$M_q = \mathrm{diag.}(m^0_{q_1}, m^0_{q_2}, m^0_{q_3}), \tag{3.19}$$

with

$$m^0_{q_i} = \lambda_{q_i} v/\sqrt{2}. \tag{3.20}$$

As seen from Eq. (3.8), the basis (a) is chosen naturally by eliminating the h_q coupling with a unitary transformation of the right-handed quarks:

$$h_q = \mathbf{0} \to \Delta'_{qQ} = \mathbf{0}. \tag{3.21}$$

This choice is in fact made irrespectively of the specific values of the VEV's. On the other hand, the condition,

$$f_Q \mathrm{e}^{i\phi_S} + f'_Q \mathrm{e}^{-i\phi_S} = \mathbf{0} \to \Delta_{qQ} = \mathbf{0}, \tag{3.22}$$

for the basis (b) seems to require some tuning between f_Q and f'_Q, which depends on the phase ϕ_S of $\langle S \rangle$. This tuning may, however, be evaded in some cases including one real S, no S and one supersymmetric S models. In these models, the f'_Q coupling is absent, and then the Δ_{qQ} term is rotated out naturally together with the f_Q coupling.

The condition $M_q = \mathbf{0}$ may even be realized by means of the symmetries and matter contents so as to distinguish the ordinary quarks from the singlet quarks. This is in fact the case in some left-right gauge models. Then, the so-called seesaw mechanism is available for generating the ordinary quark masses with three singlet quarks [31, 32, 33, 34, 35, 36, 37, 38, 39]:

$$\textbf{seesaw} \; : \; M_q = \mathbf{0} \; (N_Q = 3). \tag{3.23}$$

The seesaw case may formally be reduced to the basis (a) by the exchange $q^c \leftrightarrow Q^c$ of the right-handed quarks. It will, however, be appropriate to treat separately the seesaw case in its own right.

In the following, we examine accurately for the respective bases how the quark masses and mixings are affected by the q-Q mixing. The technical details in diagonalizing $\mathcal{M}_\mathcal{Q}$ are presented in Appendix A.

3.3.2 Basis (a) with $\Delta'_{qQ} = \mathbf{0}$

In the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, the quark mass matrix is given by

$$\mathcal{M}_\mathcal{Q} = \begin{pmatrix} M_q & \Delta_{qQ} \\ \mathbf{0} & M_Q \end{pmatrix} . \tag{3.24}$$

Then, it is relevant to introduce the q-Q mixing parameters

$$\begin{aligned} \epsilon_i^f &= \max\left[|(\Delta_{qQ})_{ia}|/m_Q\right] \\ &\sim (v_S/m_Q)(|(f_Q)_{ia}| + |(f'_Q)_{ia}|), \end{aligned} \tag{3.25}$$

each of which represents the magnitude of the mixing between the i-th ordinary quark and the singlet quarks. Here, it is assumed that the singlet quarks have masses $\sim m_Q$ of the same order.

The ordinary quark masses are obtained as

$$m_{q_i} = c_i(\boldsymbol{\epsilon}^f)\lambda_{q_i} v, \tag{3.26}$$

where $c_i(\boldsymbol{\epsilon}^f) \sim 1$ depending on $\boldsymbol{\epsilon}^f \equiv (\epsilon_1^f, \epsilon_2^f, \epsilon_3^f)$. (See Appendix A for the detailed arguments.) The q-Q mixing and the ordinary quark mixing are estimated in magnitude as

$$(\epsilon_{q_\mathrm{L}})_{ia} \sim (\epsilon'_{q_\mathrm{L}})_{ia} \sim (m_{q_i}/m_Q)\epsilon_i^f, \tag{3.27}$$

$$(\epsilon_{q_\mathrm{R}})_{ia} \sim (\epsilon'_{q_\mathrm{R}})_{ia} \sim \epsilon_i^f, \tag{3.28}$$

$$(V_{q_\mathrm{L}})_{ij} \sim \delta_{ij} + \frac{m_{q_i} m_{q_j}}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^f \epsilon_j^f, \tag{3.29}$$

$$(V_{q_\mathrm{R}})_{ij} \sim \delta_{ij} + \frac{m_{q_j}^2}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^f \epsilon_j^f, \tag{3.30}$$

where

$$\frac{m_{q_i} m_{q_j}}{m_{q_i}^2 + m_{q_j}^2} \approx \begin{cases} m_{q_i}/m_{q_j} & (m_{q_i} \ll m_{q_j}) \\ m_{q_j}/m_{q_i} & (m_{q_i} \gg m_{q_j}) \end{cases}, \tag{3.31}$$

$$\frac{m_{q_j}^2}{m_{q_i}^2 + m_{q_j}^2} \approx \begin{cases} 1 & (m_{q_i} \ll m_{q_j}) \\ (m_{q_j}/m_{q_i})^2 & (m_{q_i} \gg m_{q_j}) \end{cases}. \tag{3.32}$$

The corrections to the diagonal components of V_{q_χ} are estimated precisely in terms of the q-Q mixing matrices by noting the unitarity relations for the whole transformation matrices $\mathcal{V}_{\mathcal{Q}_\chi}$ given in Eq. (3.15):

$$V_{q_\chi}^\dagger V_{q_\chi} = \mathbf{1} - \epsilon'_{q_\chi} \epsilon'^\dagger_{q_\chi}, \tag{3.33}$$

$$V_{q_\chi} V_{q_\chi}^\dagger = \mathbf{1} - \epsilon_{q_\chi} \epsilon^\dagger_{q_\chi}. \tag{3.34}$$

We have really observed in Eq. (3.26) that the hierarchical masses m_{q_i} of the ordinary quarks are reproduced by the corresponding Yukawa couplings λ_{q_i} even in the presence of the q-Q mixing. The q-Q mixing effects are described in terms of the parameters ϵ_i^f in the basis (a) with $\Delta'_{qQ} = \mathbf{0}$. It should particularly be noted in Eq. (3.27) that the left-handed q-Q mixing is suppressed further by the q/Q mass ratios m_{q_i}/m_Q. The ordinary quark mixings $(V_{q_\chi})_{ij}$ $(i \neq j)$ arising at the order of $\epsilon_i^f \epsilon_j^f$ are actually related to the ordinary quark mass ratios in Eqs. (3.29) and (3.30). The unitarity violation of V_{q_χ} is determined by the q-Q mixing matrices ϵ_{q_χ} and ϵ'_{q_χ} in Eqs. (3.33) and (3.34). These features of the q-Q mixing effects, as seen more explicitly in the leading order calculations [5, 6, 7, 8], even hold for $N_Q \geq 1$ beyond the leading orders.

3.3.3 Basis (b) with $\Delta_{qQ} = \mathbf{0}$

In the basis (b) with $\Delta_{qQ} = \mathbf{0}$, the quark mass matrix is given by

$$\mathcal{M}_{\mathcal{Q}} = \begin{pmatrix} M_q & \mathbf{0} \\ \Delta'_{qQ} & M_Q \end{pmatrix}. \tag{3.35}$$

Then, the q-Q mixing parameters are taken as

$$\begin{aligned} \epsilon_i^h &= \max\left[|(\Delta'_{qQ})_{ai}|/m_Q \right] \\ &\sim (v/m_Q)|(h_q)_{ai}|. \end{aligned} \tag{3.36}$$

The ordinary quark masses are obtained as

$$m_{q_i} = c'_i(\boldsymbol{\epsilon}^h)\lambda_{q_i} v, \tag{3.37}$$

where $c_i'(\boldsymbol{\epsilon}^h) \sim 1$ depending on $\boldsymbol{\epsilon}^h \equiv (\epsilon_1^h, \epsilon_2^h, \epsilon_3^h)$. The q-Q mixing effects are estimated as

$$
\begin{aligned}
(\epsilon_{q_{\rm L}})_{ia} &\sim (\epsilon'_{q_{\rm L}})_{ia} \sim \epsilon_i^h, &(3.38)\\
(\epsilon_{q_{\rm R}})_{ia} &\sim (\epsilon'_{q_{\rm R}})_{ia} \sim (m_{q_i}/m_Q)\epsilon_i^h, &(3.39)\\
(V_{q_{\rm L}})_{ij} &\sim \delta_{ij} + \frac{m_{q_j}^2}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^h\epsilon_j^h, &(3.40)\\
(V_{q_{\rm R}})_{ij} &\sim \delta_{ij} + \frac{m_{q_i}m_{q_j}}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^h\epsilon_j^h. &(3.41)
\end{aligned}
$$

Here, the left-handed q-Q mixing in Eq. (3.38) is no longer suppressed by the q/Q mass ratios in contrast to Eq. (3.27) for the basis (a). Then, as described explicitly in the next section, the CKM unitarity violation and the Z mediated FCNC's can be significant for reasonable values of the q-Q mixing parameters ϵ_i^h [9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

We find again in Eq. (3.37) that the actual masses of the ordinary quarks are reproduced in terms of the relevant Yukawa couplings. It should, however, be mentioned that this relation might not be stable generally. In fact, the quark basis (b) is found by eliminating the q^cQ term $\Delta_{qQ} = f_Q\langle S\rangle + f_Q'\langle S\rangle^*$. This seems to be a fine tuning between the couplings f_Q and f_Q' with a given $\langle S\rangle$. If the complex phase of $\langle S\rangle$ is changed by radiative corrections, the quark basis (b) is rearranged so as to keep the condition $\Delta_{qQ} = \mathbf{0}$. Accordingly, the new diagonal coupling $\tilde{\lambda}_q$ is obtained as a mixture of λ_q and h_q. Then, even if the original couplings λ_{q_i} are chosen so as to reproduce the hierarchical quark masses $m_{q_i} \sim \lambda_{q_i} v$, there is in general no guarantee that the new couplings $\tilde{\lambda}_{q_i}$ also take the similar values. This problem encountered in the basis (b) may, however, be evaded reasonably in certain cases. For example, in the models without the f_Q' coupling, the f_Q coupling is eliminated from the beginning by using the right-handed quark transformation. It is also considered that the λ_q and h_q couplings have the same flavor structure as $(h_q)_{ai} \sim \lambda_{q_i} \sim m_{q_i}/v$. This relation between λ_q and h_q is in fact technically natural, and the new $\tilde{\lambda}_q$ also has the same hierarchical flavor structure.

3.3.4 Seesaw model

The quark mass matrix of the seesaw form ($N_Q = 3$) [31, 32, 33, 34, 35, 36, 37, 38, 39] is given by

$$
\mathcal{M}_\mathcal{Q} = \begin{pmatrix} \mathbf{0} & \Delta_{qQ} \\ \Delta'_{qQ} & M_Q \end{pmatrix}. \qquad (3.42)
$$

We first consider the case where the singlet quarks have comparable masses,

$$m_{Q_1},\ m_{Q_2},\ m_{Q_3} \sim m_Q. \tag{3.43}$$

Then, the ordinary quark masses are given by

$$m_{q_i} = c''_i(\boldsymbol{\epsilon}^f, \boldsymbol{\epsilon}^h)\epsilon^f_i \epsilon^h_i m_Q, \tag{3.44}$$

where $c''_i(\boldsymbol{\epsilon}^f, \boldsymbol{\epsilon}^h) \sim 1$. The q-Q mixing effects are estimated as

$$(\epsilon_{q_{\rm L}})_{ia} \sim (\epsilon'_{q_{\rm L}})_{ia} \sim \epsilon^h_i, \tag{3.45}$$

$$(\epsilon_{q_{\rm R}})_{ia} \sim (\epsilon'_{q_{\rm R}})_{ia} \sim \epsilon^f_i, \tag{3.46}$$

$$(V_{q_{\rm L}})_{ij} \sim \delta_{ij} + \frac{\epsilon^h_i \epsilon^h_j}{(\epsilon^h_i)^2 + (\epsilon^h_j)^2}, \tag{3.47}$$

$$(V_{q_{\rm R}})_{ij} \sim \delta_{ij} + \frac{\epsilon^f_i \epsilon^f_j}{(\epsilon^f_i)^2 + (\epsilon^f_j)^2}. \tag{3.48}$$

The actual masses of the ordinary quarks may be reproduced in Eq. (3.44) under the hierarchy for the q-Q mixing terms,

$$|(\Delta_{qQ})_{1a}| \ll |(\Delta_{qQ})_{2b}| \ll |(\Delta_{qQ})_{3c}| \tag{3.49}$$

and/or

$$|(\Delta'_{qQ})_{a1}| \ll |(\Delta'_{qQ})_{b2}| \ll |(\Delta'_{qQ})_{c3}|. \tag{3.50}$$

Then, by considering some typical cases, the quark mixings provided by the singlet quarks may be related to the quark masses with Eq. (3.44):

$$\text{(i)} : \epsilon^f_i \sim \epsilon^h_i \sim \sqrt{m_{q_i}/m_Q},$$

$$(V_{q_{\rm L}})_{ij} \sim (V_{q_{\rm R}})_{ij} \sim \frac{\sqrt{m_{q_i} m_{q_j}}}{m_{q_i} + m_{q_j}}. \tag{3.51}$$

$$\text{(ii)} : \epsilon^f_i \sim m_{q_i}/\bar{h}v,\ \epsilon^h_i \sim \bar{h}v/m_Q,$$

$$(V_{q_{\rm L}})_{ij} \sim 1,\ (V_{q_{\rm R}})_{ij} \sim \frac{m_{q_i} m_{q_j}}{m^2_{q_i} + m^2_{q_j}}. \tag{3.52}$$

$$\text{(iii)} : \epsilon^f_i \sim \bar{f}v_S/m_Q,\ \epsilon^h_i \sim m_{q_i}/\bar{f}v_S,$$

$$(V_{q_{\rm L}})_{ij} \sim \frac{m_{q_i} m_{q_j}}{m^2_{q_i} + m^2_{q_j}},\ (V_{q_{\rm R}})_{ij} \sim 1. \tag{3.53}$$

In the case (i) the q-Q mixing terms Δ_{qQ} and Δ'_{qQ} are assumed to have the same flavor structure. In the case (ii) only the Δ_{qQ} term has the hierarchical form (3.49) while the

Δ'_{qQ} term $\sim \bar{h}v$ is rather flavor-independent. On the other hand, in the case (iii) only the Δ'_{qQ} term has the hierarchical form (3.50) while the Δ_{qQ} term $\sim \bar{f}v_S$ is rather flavor-independent.

While the above choice for the model parameters is technically natural, there is another attractive possibility for realizing the ordinary quark mass hierarchy [39]. That is, the inverted hierarchy is assumed for the singlet quark masses,

$$m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}. \tag{3.54}$$

The specific relations such as Eqs. (3.49) and (3.50) are, however, not invoked on the q-Q mixing terms, i.e.,

$$(\Delta_{qQ})_{ia} \sim \bar{f}v_S,\ (\Delta'_{qQ})_{ai} \sim \bar{h}v, \tag{3.55}$$

where $\bar{f}$ and $\bar{h}$ represent the mean values of the relevant Yukawa couplings. In this situation, the ordinary quark masses are given by

$$m_{q_i} = c_i'''(\bar{f},\bar{h})(\bar{f}v_S/m_{Q_i})\bar{h}v, \tag{3.56}$$

where $c_i'''(\bar{f},\bar{h}) \sim 1$ [39]. Accordingly, the quark mixings are obtained in terms of the ordinary quark masses as

$$(\tilde{V}^\dagger_{q_{\mathrm{L}}}\epsilon_{q_{\mathrm{L}}})_{ia} \sim (\epsilon'_{q_{\mathrm{L}}})_{ia} \sim (m_{q_i}/\bar{f}v_S), \tag{3.57}$$

$$(\tilde{V}^\dagger_{q_{\mathrm{R}}}\epsilon_{q_{\mathrm{R}}})_{ia} \sim (\epsilon'_{q_{\mathrm{R}}})_{ia} \sim (m_{q_i}/\bar{h}v), \tag{3.58}$$

$$(\tilde{V}^\dagger_{q_{\mathrm{L}}}V_{q_{\mathrm{L}}})_{ij} \sim \frac{m_{q_i}m_{q_j}}{m_{q_i}^2+m_{q_j}^2}, \tag{3.59}$$

$$(\tilde{V}^\dagger_{q_{\mathrm{R}}}V_{q_{\mathrm{R}}})_{ij} \sim \frac{m_{q_i}m_{q_j}}{m_{q_i}^2+m_{q_j}^2}. \tag{3.60}$$

Here, the unitary matrices $\tilde{V}^\dagger_{q_{\mathrm{L}}}$ and $\tilde{V}^\dagger_{q_{\mathrm{R}}}$ are introduced to deform the Δ_{qQ} and Δ'_{qQ} terms to the triangular forms as given in Eqs. (3.184) and (3.185).

It is indeed seen in Eq. (3.56) that the ordinary quark mass hierarchy is realized by the inverted hierarchy (3.54) of singlet quarks. These relations for the quark masses and mixings are essentially applicable even for the case $m_T^0 \ll \bar{f}v_S$ (m_T^0 is the third component of the diagonal M_U), as mentioned in Ref. [39]. In this case, we obtain

$$m_t \sim \bar{h}v,\ m_T \sim \bar{f}v_S \tag{3.61}$$

with the significant right-handed t-T mixing with $(\epsilon_{q_{\mathrm{R}}})_{tT}, (\epsilon'_{q_{\mathrm{R}}})_{tT} \simeq 1$.

3.3.5 Relations among the three bases

We have described so far the structures of the q-Q mixing effects on the quark masses and mixings. They are summarized in Table 3.1. We here discuss the relations among these three representative bases. The basis (a) with $\Delta'_{qQ} = \mathbf{0}$ is suitable as long as the Δ_{qQ} term does not exceed the M_Q term. In this case, the quark masses and mixings are described well in terms of $\epsilon_i^f \sim |(\Delta_{qQ})_{ia}|/m_Q \lesssim 1$. This treatment is, however, invalidated when $|\Delta_{qQ}| \gg |M_Q|$ in the basis (a). Then, we may seek a more appropriate quark basis by making some unitary transformations of the quarks. These transformations should respect the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, i.e., the components of the doublets are not mixed with the singlets in the left-handed quark sector before the quark mass terms are generated by the spontaneous breakdown of the $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$.

In general, for any $N_Q \geq 1$ the quark mass matrix in the basis (a) can be deformed as follows by a unitary transformation $\tilde{\mathcal{V}}^{(1)}_{\mathcal{Q}_\mathrm{R}}$ of the right-handed quarks:

$$\tilde{\mathcal{V}}^{(1)\dagger}_{\mathcal{Q}_\mathrm{R}} \begin{pmatrix} M_q & \Delta_{qQ} \\ \mathbf{0} & M_Q \end{pmatrix} = \begin{pmatrix} \tilde{M}^{(1)}_q & \mathbf{0} \\ \tilde{\Delta}^{(1)\prime}_{qQ} & \tilde{M}^{(1)}_Q \end{pmatrix}. \tag{3.62}$$

The q-Q mixing in $\tilde{\mathcal{V}}^{(1)}_{\mathcal{Q}_\mathrm{R}}$ is significant with $\tilde{\epsilon}^{(1)}_{q_\mathrm{R}} \sim 1$ for the case of $|\Delta_{qQ}| \gtrsim |M_Q|$. Here, we note the relation between $\tilde{M}^{(1)}_q$ and $\tilde{\Delta}^{(1)\prime}_{qQ}$ both of which are provided by the original M_q:

$$\tilde{\Delta}^{(1)\prime}_{qQ} = \tilde{\epsilon}^{(1)\dagger}_{q_\mathrm{R}} \tilde{V}^{(1)\dagger -1}_{q_\mathrm{R}} \tilde{M}^{(1)}_q. \tag{3.63}$$

The mass matrix $\tilde{M}^{(1)}_q$ is then diagonalized by the unitary transformations $\tilde{V}^{(2)}_{q_\mathrm{L}}$ and $\tilde{V}^{(2)}_{q_\mathrm{R}}$ of the ordinary quarks as

$$\begin{aligned} \tilde{M}_q &= \tilde{V}^{(2)\dagger}_{q_\mathrm{R}} \tilde{M}^{(1)}_q \tilde{V}^{(2)}_{q_\mathrm{L}} \\ &= \mathrm{diag.}(\tilde{m}^0_{q_1}, \tilde{m}^0_{q_2}, \tilde{m}^0_{q_3}). \end{aligned} \tag{3.64}$$

Accordingly, the $\tilde{\Delta}^{(1)\prime}_{qQ}$ term is deformed as

$$\tilde{\Delta}'_{qQ} = \tilde{\Delta}^{(1)\prime}_{qQ} \tilde{V}^{(2)}_{q_\mathrm{L}} = \tilde{\epsilon}^{(1)\dagger}_{q_\mathrm{R}} \tilde{V}^{(1)\dagger -1}_{q_\mathrm{R}} \tilde{V}^{(2)}_{q_\mathrm{R}} \tilde{M}_q. \tag{3.65}$$

In this way, starting with the quark mass matrix $\mathcal{M}_\mathcal{Q}$ in the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, we have obtained the quark mass matrix $\tilde{\mathcal{M}}_\mathcal{Q}$ in the basis (b) with $\tilde{\Delta}_{qQ} = \mathbf{0}$.

quark mass matrix form $\mathcal{M}_{\mathcal{Q}}$		ordinary quark masses m_{q_i}	q-Q mixings [left-handed] $(\epsilon_{q_\mathrm{L}})_{ia},\ (\epsilon'_{q_\mathrm{L}})_{ia}$	[right-handed] $(\epsilon_{q_\mathrm{R}})_{ia},\ (\epsilon'_{q_\mathrm{R}})_{ia}$	ordinary quark mixings [left-handed] $(V_{q_\mathrm{L}})_{ij}(i \neq j)$	[right-handed] $(V_{q_\mathrm{R}})_{ij}(i \neq j)$
basis (a) : $\Delta'_{qQ} = \mathbf{0}$		$\lambda_{q_i} v$	$\dfrac{m_{q_i}}{m_Q}\epsilon_i^f$	ϵ_i^f	$\dfrac{m_{q_i} m_{q_j}}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^f \epsilon_j^f$	$\dfrac{m_{q_j}^2}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^f \epsilon_j^f$
basis (b) : $\Delta_{qQ} = \mathbf{0}$		$\lambda_{q_i} v$	ϵ_i^h	$\dfrac{m_{q_i}}{m_Q}\epsilon_i^h$	$\dfrac{m_{q_j}^2}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^h \epsilon_j^h$	$\dfrac{m_{q_i} m_{q_j}}{m_{q_i}^2 + m_{q_j}^2}\epsilon_i^h \epsilon_j^h$
seesaw : $M_q = \mathbf{0}$	[com]	$\epsilon_i^f \epsilon_i^h m_Q$	ϵ_i^h	ϵ_i^f	$\dfrac{\epsilon_i^h \epsilon_j^h}{(\epsilon_i^h)^2 + (\epsilon_j^h)^2}$	$\dfrac{\epsilon_i^f \epsilon_j^f}{(\epsilon_i^f)^2 + (\epsilon_j^f)^2}$
	[inv]	$\dfrac{\bar{f} v_S}{m_{Q_i}}\bar{h} v$	$\dfrac{m_{q_i}}{\bar{f} v_S}$	$\dfrac{m_{q_i}}{\bar{h} v}$	$\dfrac{m_{q_i} m_{q_j}}{m_{q_i}^2 + m_{q_j}^2}$	$\dfrac{m_{q_i} m_{q_j}}{m_{q_i}^2 + m_{q_j}^2}$

Table 3.1: The ordinary quark masses and mixings are listed for the representative bases, which are obtained in the presence of singlet quarks. For the seesaw model, [com] indicates the case of comparable singlet quark masses $m_{Q_1}, m_{Q_2}, m_{Q_3} \sim m_Q$, and [inv] the case of inverted hierarchy $m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}$.

Here, it is noticed in Eq. (3.65) that the $\tilde{\Delta}'_{qQ}$ term is related to the ordinary quark mass matrix $\tilde{M}_q$ as

$$(\tilde{\Delta}'_{qQ})_{ai} \sim (\tilde{M}_q)_{ii}. \tag{3.66}$$

This implies the relation of the Yukawa couplings,

$$(\tilde{h}_q)_{ai} \sim \tilde{\lambda}_{q_i}. \tag{3.67}$$

Then, the quark masses are obtained as $m_{q_i} \sim \tilde{m}^0_{q_i} = (\tilde{M}_q)_{ii}$, and the q-Q mixing parameters are specifically given by the q/Q mass ratios as

$$\tilde{\epsilon}^h_i \sim |(\tilde{\Delta}'_{qQ})_{ai}|/m_Q \sim m_{q_i}/m_Q. \tag{3.68}$$

By considering these arguments, the case of the basis (a) may be regarded as a special case of the basis (b). It is, in particular, interesting that the specific relation (3.67) for the Yukawa couplings, which may be invoked ad hoc in the basis (b), is obtained naturally starting from the basis (a).

We next consider the cases with $N_Q = 3$ including the seesaw model. The quark mass matrix in the basis (a) can be deformed to the seesaw form by the exchange $q^c \leftrightarrow Q^c$:

$$\begin{pmatrix} M_q & \Delta_{qQ} \\ \mathbf{0} & M_Q \end{pmatrix} \to \begin{pmatrix} \mathbf{0} & M_Q \\ M_q & \Delta_{qQ} \end{pmatrix}. \tag{3.69}$$

Then, if $|\Delta_{qQ}| \gtrsim |M_Q|$, the relations for the q-Q mixing effects in the seesaw case can be applied. We have also seen that if $|\Delta_{qQ}| \gtrsim |M_Q|$ in the basis (a), it is appropriate to transfer from the basis (a) to the basis (b). Hence, it is expected that the seesaw case is even related to the case of basis (b). In fact, the seesaw quark mass matrix can be deformed as

$$\begin{pmatrix} \mathbf{0} & \Delta_{qQ} \\ \Delta'_{qQ} & M_Q \end{pmatrix} \to \begin{pmatrix} \tilde{M}_q & \mathbf{0} \\ \tilde{\Delta}'_{qQ} & \tilde{M}_Q \end{pmatrix}. \tag{3.70}$$

Here, the Δ_{qQ} term is first eliminated by the right-handed quark transformation, and then the $\tilde{M}_q$ term is made diagonal by the ordinary quark transformation. The relation between $\tilde{\Delta}'_{qQ}$ and $\tilde{M}_q$, which is similar to Eq. (3.65), is obtained since they both stem from the Δ'_{qQ} term.

If we start with a quark basis with general form of $\mathcal{M}_{\mathcal{Q}}$, the quark masses and mixings are apparently given as a mixture of the three representative cases. The quark masses are obtained as

$$m_{q_i} \sim \lambda_{q_i} v + \epsilon^f_i \epsilon^h_i m_Q.$$

Then, the hierarchy of the ordinary quark masses may be reproduced by the tuning between these two contributions or by relating the q-Q mixing parameters ϵ_i^f and/or ϵ_i^h to the corresponding Yukawa couplings λ_{q_i}. The latter choice seems to be technically natural. The q-Q mixing effects on the ordinary quark mixing are mainly described in terms of the ratios $\epsilon_i^f/\epsilon_j^f$ and $\epsilon_i^h/\epsilon_j^h$, as given in Eqs. (3.47) and (3.48) for the seesaw case. It should anyway be remarked that these q-Q mixing effects are reduced to those given in one of the three representative bases, if appropriate rearrangements are made for the quark fields and relevant Yukawa couplings.

To summarize, we may take complementarily the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, the basis (b) with $\Delta_{qQ} = \mathbf{0}$ and the seesaw with $M_q = \mathbf{0}$. Some choice of the basis may appear to be more suitable than the others, depending on the parameter ranges and also the symmetries and matter contents. For example, the seesaw form mass matrix is obtained readily in the left-right gauge models [31, 32, 33, 34, 35, 36, 37, 38, 39]. The basis (b) will be suitable if $|\Delta_{qQ}| \gg |M_Q|$ in the basis (a). On the other hand, the ad hoc relation (3.67) in the basis (b) is realized naturally starting from the basis (a). In the model with complex singlet Higgs field S, the condition $\Delta_{qQ} = \mathbf{0}$ in the basis (b) might require a tuning between the f_Q and f'_Q couplings, as seen in Eq. (3.22), unless $f'_Q \equiv \mathbf{0}$ is ensured by means of certain symmetry. In any case, by taking the appropriate quark basis we can find the reasonable regions of the model parameter space where the actual masses of the ordinary quarks are reproduced even in the presence of singlet quarks.

3.4 Flavor changing interactions

In this section, we examine the flavor changing interactions of quarks which are affected by the q-Q mixing. Specifically, the CKM unitarity within the ordinary quark sector is violated, and the FCNC's arise both in the gauge and scalar couplings [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. We would like to clarify the structures of these flavor changing interactions for the representative bases. In fact, they are described appropriately in terms of the q-Q mixing parameters and the quark masses. Hence, the present model with singlet quarks may provide an interesting extension of the notion of natural flavor conservation [29, 30].

3.4.1 Charged currents

The charged gauge interaction coupled to the W boson is expressed in terms of the quark mass eigenstates as

$$\begin{aligned} \mathcal{L}_{\rm CC}(W) &= g W_\mu^+ \mathcal{U}^\dagger \sigma^\mu \mathcal{V} \mathcal{D} + \text{H.c.} \\ &= g W_\mu^+ u^\dagger \sigma^\mu V d + \cdots . \end{aligned} \tag{3.71}$$

Here the generalized left-handed quark mixing matrix for the charged weak currents is given by

$$\mathcal{V} = \mathcal{V}_{\mathcal{U}_{\rm L}}^\dagger \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{D}_{\rm L}} . \tag{3.72}$$

The CKM matrix V for the ordinary quarks, which is included in $\mathcal{V}$ as a submatrix, is actually modified from the original V_0 as

$$V = V_{u_{\rm L}}^\dagger V_0 V_{d_{\rm L}} . \tag{3.73}$$

Here, $V_{u_{\rm L}}$ and $V_{d_{\rm L}}$ are the left-handed ordinary quark mixings induced by the q-Q mixing, which are presented in the previous section.

In the present model containing only one Higgs doublet, there is no physical charged Higgs particle mediating the scalar interactions of quarks. If supersymmetric models with a pair of Higgs doublets H_1 and H_2 are considered, one physical charged scalar particle appears. Then, this charged scalar particle as well as the Nambu-Goldstone mode absorbed by the W boson have the Yukawa couplings which are described in terms of the quark masses and CKM matrix as

$$\Lambda_u^+ = (\bar{M}_u / \langle H_2^0 \rangle) V, \ \Lambda_d^- = (\bar{M}_d / \langle H_1^0 \rangle) V^\dagger . \tag{3.74}$$

Therefore, these charged scalar couplings with the same CKM structure as the charged gauge interaction do not provide so distinct effects on the flavor changing processes.

The unitarity violation of the CKM matrix V is calculated with the unitarity relations (3.33) and (3.34) of $\mathcal{V}_{\mathcal{Q}_{\rm L}}$:

$$V^\dagger V - \mathbf{1} = -\epsilon'_{d_{\rm L}} \epsilon'^\dagger_{d_{\rm L}} - V_{d_{\rm L}}^\dagger V_0^\dagger \epsilon_{u_{\rm L}} \epsilon_{u_{\rm L}}^\dagger V_0 V_{d_{\rm L}} , \tag{3.75}$$

$$V V^\dagger - \mathbf{1} = -\epsilon'_{u_{\rm L}} \epsilon'^\dagger_{u_{\rm L}} - V_{u_{\rm L}}^\dagger V_0 \epsilon_{d_{\rm L}} \epsilon_{d_{\rm L}}^\dagger V_0^\dagger V_{u_{\rm L}} . \tag{3.76}$$

As shown in the next subsection, the modification of the Z mediated neutral currents as well as this CKM unitarity violation are described in terms of the second order q-Q mixing factors $\epsilon_{q_L}\epsilon_{q_L}^\dagger$ and $\epsilon'_{q_L}\epsilon'^\dagger_{q_L}$ [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. In the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, the CKM unitarity violation is actually far below the experimental bounds [1]. This is due to the fact that the left-handed q-Q mixing is suppressed by the q/Q mass ratios, as seen in Eq. (3.27). On the other hand, if the model parameters are taken so that the basis (b) with $\Delta_{qQ} = \mathbf{0}$ is relevant, the q-Q mixing effects on the flavor changing interactions are not necessarily suppressed by the q/Q mass ratios. Hence, they can be comparable to the current experimental bounds, as usually considered in the literature [9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. As for the seesaw model, the q-Q mixing parameters may be related to the ordinary quark masses, as discussed in the previous section. Then, the q-Q mixing effects are substantially suppressed.

We now consider how the CKM structure with small flavor changing elements is reproduced in the presence of q-Q mixing. We have seen that in some cases the left-handed ordinary quark mixing V_{q_L} induced by the q-Q mixing is related to the ordinary quark mass hierarchy. Then, V_{q_L} is close enough to the unit matrix, and the actual CKM matrix V can be obtained readily by taking $V_0 \simeq V$. In other cases, as seen for example in Eq. (3.52), V_{q_L} itself may deviate significantly from the unit matrix. Even in such cases, the unitarity violation of V_{q_L} arising at the second order of the q-Q mixing parameters are constrained to be small enough phenomenologically. Then, the realistic CKM matrix can be reproduced by taking $V_0 \simeq V_{u_L} V V_{d_L}^\dagger$. As long as the mixing matrix V_0 can be taken freely, it is in fact impossible to make some definite predictions on the CKM matrix. It is, however, at least technically natural that the actual CKM matrix is close enough to the unit matrix. This choice for the CKM matrix can be ensured by means of the approximate flavor symmetries, as is the case in the minimal standard model. It will be an interesting possibility, as considered in the seesaw models for quark masses [31, 32, 33, 34, 35, 36, 37], that some predictions on the CKM matrix are obtained by invoking some global chiral symmetry to restrict the forms of Yukawa coupling matrices.

3.4.2 Neutral currents

We next describe the neutral currents of quarks coupled to the Z boson and Higgs scalar particles, which are also modified by the q-Q mixing.

Neutral gauge couplings

The neutral gauge interaction of quarks mediated by the Z boson is given by

$$\mathcal{L}_{\rm NC}(Z) = \frac{g}{\cos\theta_W} Z_\mu J_Z^\mu \tag{3.77}$$

with

$$J_Z^\mu = \sum_{\mathcal{U},\mathcal{D}} \mathcal{Q}^\dagger \sigma^\mu \mathcal{Z}_\mathcal{Q} \mathcal{Q} \ + \sum_{\mathcal{U}^c,\mathcal{D}^c} \mathcal{Q}^{c\dagger} \sigma^\mu \mathcal{Z}_{\mathcal{Q}^c} \mathcal{Q}^c. \tag{3.78}$$

The coupling matrices are given by

$$\begin{aligned} \mathcal{Z}_\mathcal{Q} &= \mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \mathcal{Z}_\mathcal{Q}^0 \mathcal{V}_{\mathcal{Q}_{\rm L}}, & (3.79)\\ \mathcal{Z}_{\mathcal{Q}^c} &= I_Z(q_0^c) \left(\begin{array}{cc} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{array} \right), & (3.80) \end{aligned}$$

where

$$\mathcal{Z}_\mathcal{Q}^0 = \left(\begin{array}{cc} I_Z(q_0)\mathbf{1} & \mathbf{0} \\ \mathbf{0} & I_Z(Q_0)\mathbf{1} \end{array} \right), \tag{3.81}$$

and

$$I_Z(\mathcal{F}) = I_3(\mathcal{F}) - \sin^2\theta_W Q_{\rm em}(\mathcal{F}) \tag{3.82}$$

for $\mathcal{F} = q_0, q_0^c, Q_0, Q_0^c$.

The right-handed couplings are unchanged with the diagonal and flavor-universal form for $I_3(q_0^c) = I_3(Q_0^c) = 0$. The variation of the left-handed couplings induced by the q-Q mixing is calculated as

$$\begin{aligned} \Delta\mathcal{Z}_\mathcal{Q} &\equiv \mathcal{Z}_\mathcal{Q} - \mathcal{Z}_\mathcal{Q}^0 \\ &= I_3(q_0) \left(\begin{array}{cc} -\epsilon'_{q_{\rm L}} \epsilon'^\dagger_{q_{\rm L}} & V^\dagger_{q_{\rm L}} \epsilon_{q_{\rm L}} \\ \epsilon^\dagger_{q_{\rm L}} V_{q_{\rm L}} & \epsilon^\dagger_{q_{\rm L}} \epsilon_{q_{\rm L}} \end{array} \right). \end{aligned} \tag{3.83}$$

The upper-left component, say $\Delta\mathcal{Z}_\mathcal{Q}[q]$, in particular, describes the neutral currents among the ordinary quarks:

$$\Delta\mathcal{Z}_\mathcal{Q}[q] = -I_3(q_0) \epsilon'_{q_{\rm L}} \epsilon'^\dagger_{q_{\rm L}}. \tag{3.84}$$

It should here be noticed that this variation of the neutral currents (3.84) as well as the CKM unitarity violation given in Eqs. (3.75) and (3.76) arise at the second order of q-Q mixing [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

In the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, the variation of the Z mediated neutral currents induced by the q-Q mixing is estimated with Eq. (3.27) as

$$\Delta\mathcal{Z}_\mathcal{Q}[q]_{ij} \sim (m_{q_i}/m_Q)(m_{q_j}/m_Q) \epsilon_i^f \epsilon_j^f. \tag{3.85}$$

This correction as well as the CKM unitarity violation are suppressed substantially by the second order of q/Q mass ratios, providing negligible effects except for those involving the top quark.

On the other hand, in the basis (b) with $\Delta_{qQ} = \mathbf{0}$ we obtain

$$\Delta \mathcal{Z}_{\mathcal{Q}}[q]_{ij} \sim \epsilon^h_i \epsilon^h_j. \tag{3.86}$$

This q-Q mixing effect on the Z mediated neutral currents as well as the CKM unitarity violation are no longer suppressed by the q/Q mass ratios. Then, some meaningful constraints are placed phenomenologically on the q-Q mixing, and such constraints provide restrictions on the possible contributions to the flavor changing processes [9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

The modifications of the neutral gauge couplings as given in Eq. (3.86) for the basis (b) are also obtained for the seesaw model with $m_{Q_1}, m_{Q_2}, m_{Q_3} \sim m_Q$. Then, for the typical cases considered in Eqs. (3.51), (3.52) and (3.53), they are estimated as

$$\text{(i)} \quad : \quad \Delta \mathcal{Z}_{\mathcal{Q}}[q]_{ij} \sim \sqrt{(m_{q_i}/m_Q)(m_{q_j}/m_Q)}, \tag{3.87}$$

$$\text{(ii)} \quad : \quad \Delta \mathcal{Z}_{\mathcal{Q}}[q]_{ij} \sim (\bar{h}v/m_Q)^2, \tag{3.88}$$

$$\text{(iii)} \quad : \quad \Delta \mathcal{Z}_{\mathcal{Q}}[q]_{ij} \sim (m_{q_i}/\bar{f}v_S)(m_{q_j}/\bar{f}v_S). \tag{3.89}$$

Here, for the case (i) we have estimates with the ordinary quark masses as

$$\Delta \mathcal{Z}_{\mathcal{U}}[u] \sim \begin{pmatrix} 10^{-5} & 10^{-4} & 10^{-3} \\ 10^{-4} & 10^{-3} & 10^{-2} \\ 10^{-3} & 10^{-2} & 10^{-1} \end{pmatrix} \frac{500\mathrm{GeV}}{m_U}, \tag{3.90}$$

$$\Delta \mathcal{Z}_{\mathcal{D}}[d] \sim \begin{pmatrix} 10^{-5} & 10^{-5} & 10^{-4} \\ 10^{-5} & 10^{-4} & 10^{-2} \\ 10^{-4} & 10^{-2} & 10^{-2} \end{pmatrix} \frac{500\mathrm{GeV}}{m_D}. \tag{3.91}$$

These q-Q mixing effects in the case (i) can be comparable to the experimental bounds [1] for $m_Q \sim 500$GeV. In the case (ii), where the Z mediated FCNC's are rather flavor-independent, some stringent constraints will be placed phenomenologically on the mass ratio $\bar{h}v/m_Q$.

As for the seesaw model with $m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}$, where the left-handed q-Q mixing is given by Eq. (3.57), the Z mediated FCNC's appear to be the same as given in Eq. (3.89) for the case (iii). They are fairly suppressed by the second order of the ordinary quark masses, which is also the case in the basis (a).

Neutral scalar couplings

The neutral scalar couplings of quarks are extracted from Eq. (3.1) as

$$\mathcal{L}_{\rm NC}(\phi) = -\frac{1}{\sqrt{2}} \sum_{\alpha=1,2,3} \mathcal{Q}^c \Lambda^\alpha_\mathcal{Q} \mathcal{Q} \phi_\alpha \ + \ \text{H.c.}, \tag{3.92}$$

where ϕ_1, ϕ_2, ϕ_3 represent the mass eigenstates of the neutral scalar fields. The coupling matrices are given by

$$\Lambda^\alpha_\mathcal{Q} = O_{\alpha 1} \Lambda^H_\mathcal{Q} + O_{\alpha 2} \Lambda^{S_+}_\mathcal{Q} + i O_{\alpha 3} \Lambda^{S_-}_\mathcal{Q} \tag{3.93}$$

with

$$\Lambda^H_\mathcal{Q} \ = \ \mathcal{V}^\dagger_{\mathcal{Q}_{\rm R}} \begin{pmatrix} \lambda_q & \mathbf{0} \\ h_q & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}}, \tag{3.94}$$

$$\Lambda^{S_\pm}_\mathcal{Q} \ = \ \mathcal{V}^\dagger_{\mathcal{Q}_{\rm R}} \begin{pmatrix} \mathbf{0} & f_Q \pm f'_Q \\ \mathbf{0} & \lambda_Q \pm \lambda'_Q \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}}. \tag{3.95}$$

Here an orthogonal matrix O is introduced to parameterize the mass eigenstates of the neutral scalar fields:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = O \begin{pmatrix} h_1 \\ s_1 \\ s_2 \end{pmatrix}. \tag{3.96}$$

The original complex Higgs fields are decomposed with the real scalar fields as

$$H^0 \ = \ \langle H^0 \rangle + (h_1 + i h_2)/\sqrt{2}, \tag{3.97}$$

$$S \ = \ \langle S \rangle + (s_1 + i s_2)/\sqrt{2}. \tag{3.98}$$

While the Nambu-Goldstone mode h_2 is absorbed by the Z boson, the remaining h_1, s_1, s_2 are combined to form the mass eigenstates ϕ_α. At present, the masses m_{ϕ_α} and mixing matrix O of the neutral scalar fields should be regarded as free parameters varying in some reasonable range. If the hierarchy $v_S \gg v$ is realized, the mixing between the Higgs doublet and singlet will be of the order of v/v_S.

Let us examine in detail the structures of the submatrices, say $\Lambda^\alpha_\mathcal{Q}[q]$, describing the neutral scalar couplings of the ordinary quarks:

$$\Lambda^\alpha_\mathcal{Q}[q] = O_{\alpha 1} \Lambda^H_\mathcal{Q}[q] + O_{\alpha 2} \Lambda^{S_+}_\mathcal{Q}[q] + i O_{\alpha 3} \Lambda^{S_-}_\mathcal{Q}[q], \tag{3.99}$$

where

$$\Lambda^H_\mathcal{Q}[q] = \hat{\lambda}_q + \hat{h}_q, \ \Lambda^{S_\pm}_\mathcal{Q}[q] = \hat{f}^\pm_Q + \hat{\lambda}^\pm_Q \tag{3.100}$$

with

$$\hat{\lambda}_q = V_{q_\mathrm{R}}^\dagger \lambda_q V_{q_\mathrm{L}}, \tag{3.101}$$

$$\hat{h}_q = -\epsilon'_{q_\mathrm{R}} h_q V_{q_\mathrm{L}}, \tag{3.102}$$

$$\hat{f}_Q^\pm = -V_{q_\mathrm{R}}^\dagger (f_Q \pm f'_Q) \epsilon'^\dagger_{q_\mathrm{L}}, \tag{3.103}$$

$$\hat{\lambda}_Q^\pm = \epsilon'_{q_\mathrm{R}} (\lambda_Q \pm \lambda'_Q) \epsilon'^\dagger_{q_\mathrm{L}}. \tag{3.104}$$

By considering the relations for the q-Q mixing effects, which are described in Sec. 3.3, the flavor structure of the neutral scalar couplings $\Lambda_\mathcal{Q}^\alpha[q]_{ij}$ is specified for the respective bases. We suppress below for simplicity the neutral Higgs mixing parameters by assuming $O_{\alpha\beta} \sim 1$, though they are readily recovered for the scalar couplings.

We first note the relations,

$$\hat{\lambda}_q + \hat{h}_q = \Lambda_\mathcal{Q}^H[q] = (\bar{M}_q/v) V_{q_\mathrm{L}}^\dagger V_{q_\mathrm{L}}, \tag{3.105}$$

$$\hat{\lambda}_q + (v_S/v)\hat{f}_Q^S = V_{q_\mathrm{R}}^\dagger V_{q_\mathrm{R}} (\bar{M}_q/v), \tag{3.106}$$

$$(v_S/v)\hat{\lambda}_Q^S + \hat{h}_q = \epsilon'_{q_\mathrm{R}} \epsilon'^\dagger_{q_\mathrm{R}} (\bar{M}_q/v), \tag{3.107}$$

$$(v_S/v)(\hat{\lambda}_Q^S + \hat{f}_Q^S) = (\bar{M}_q/v) \epsilon'_{q_\mathrm{L}} \epsilon'^\dagger_{q_\mathrm{L}}, \tag{3.108}$$

where

$$\hat{f}_Q^S = -V_{q_\mathrm{R}}^\dagger (f_Q \mathrm{e}^{i\phi_S} + f'_Q \mathrm{e}^{-i\phi_S}) \epsilon'^\dagger_{q_\mathrm{L}}, \tag{3.109}$$

$$\hat{\lambda}_Q^S = \epsilon'_{q_\mathrm{R}} (\lambda_Q \mathrm{e}^{i\phi_S} + \lambda'_Q \mathrm{e}^{-i\phi_S}) \epsilon'^\dagger_{q_\mathrm{L}}. \tag{3.110}$$

In order to derive these relations, Eq. (3.10) is multiplied by the products of matrices,

$$\mathcal{V}_{\mathcal{Q}_\chi}^\dagger \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_\chi}, \ \mathcal{V}_{\mathcal{Q}_\chi}^\dagger \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_\chi},$$

from the right for $\chi = \mathrm{L}$ or left for $\chi = \mathrm{R}$. The relation (3.105), in particular, implies physically that the couplings of the Z boson and the Higgs field H^0 including the Nambu-Goldstone mode have the same flavor structure. In fact, by considering Eqs. (3.84) and (3.105) with Eq. (3.33) we find the relation for the FCNC's,

$$(\Lambda_\mathcal{Q}^H[q])_{ij}^{(i\neq j)} = (m_{q_i}/v)\Delta\mathcal{Z}_\mathcal{Q}[q]_{ij}/I_3(q_0). \tag{3.111}$$

Then, it is sufficient to calculate the contributions $\Lambda_\mathcal{Q}^{S_\pm}[q]$ with the singlet Higgs S.

In the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, the Z mediated FCNC's are suppressed by the second order of q/Q mass ratios. Hence, the f_Q and f'_Q couplings provide dominant contributions to the scalar FCNC's. They are estimated as

$$(\hat{f}_Q^{\pm})_{ij} = -(V_{q\mathrm{R}}^\dagger)_{ii}(f_Q^{\pm})_{ia}(\epsilon'^\dagger_{q\mathrm{L}})_{aj} - \sum_{k \neq i}(V_{q\mathrm{R}}^\dagger)_{ik}(f_Q^{\pm})_{ka}(\epsilon'^\dagger_{q\mathrm{L}})_{aj},$$

where Eqs. (3.27) and (3.30) for the quark mixings are considered. The first term amounts to the order of $(m_{q_j}/v_S)\epsilon_i^f\epsilon_j^f$. The second term is estimated as $\epsilon_i^f\epsilon_k^f\epsilon_k^f(m_Q/v_S)(m_{q_j}/m_Q)\epsilon_j^f \lesssim (m_{q_j}/v_S)\epsilon_i^f\epsilon_j^f$. Similar estimates are made for the λ_Q and λ'_Q contributions. Then, the leading contributions to the neutral scalar couplings in the basis (a) with $\Delta'_{qQ} = \mathbf{0}$ are given by

$$\Lambda_Q^\alpha[q]_{ij} \sim (m_{q_i}/v)\delta_{ij} + (m_{q_j}/v_S)\epsilon_i^f\epsilon_j^f. \tag{3.112}$$

Here, in contrast to the Z mediated FCNC's, the scalar FCNC's in the basis (a) are suppressed only by the first order of ordinary quark masses. The relevant factors are estimated as

$$\frac{m_{u_j}}{v_S} \sim \left(10^{-5}, 10^{-3}, 10^{-1}\right)\frac{500\mathrm{GeV}}{v_S}, \tag{3.113}$$

$$\frac{m_{d_j}}{v_S} \sim \left(10^{-5}, 10^{-4}, 10^{-2}\right)\frac{500\mathrm{GeV}}{v_S}. \tag{3.114}$$

Then, these scalar FCNC's are expected to provide significant phenomenological effects for $\epsilon_i^f \sim 0.1 - 1$ and $v_S \sim 100\mathrm{GeV} - 1\mathrm{TeV}$ [24, 25].

We here mention that in certain models these contributions to the scalar FCNC's substantially cancel out. For instance, suppose that $f'_Q = \mathbf{0}$ and $\lambda'_Q = \mathbf{0}$, as is the case for the one real S model and the one supersymmetric S model. Then, we have $\hat{f}_Q + \hat{\lambda}_Q = \mathrm{e}^{-i\phi_S}(\hat{f}_Q^S + \hat{\lambda}_Q^S) = (\bar{M}_q/v_S)\epsilon'_{q\mathrm{L}}\epsilon'^\dagger_{q\mathrm{L}}$ from Eq. (3.108), which is even smaller by the factor v/v_S than the Λ_Q^H contribution given in Eq. (3.105). Hence, for this specific case with $f'_Q = \mathbf{0}$ and $\lambda'_Q = \mathbf{0}$ we have the scalar FCNC's $\Lambda_Q^\alpha[q]_{ij}^{(i \neq j)} \sim (m_{q_i}/v)(m_{q_i}/m_Q)(m_{q_j}/m_Q)\epsilon_i^f\epsilon_j^f$, which are related to the Z mediated FCNC's as seen in Eq. (3.111). This result is also valid for the no singlet Higgs S model with bare Δ_{qQ} and M_Q terms. It should further be remarked for completeness that in some models only M_Q is the bare mass term, but the Δ_{qQ} term is provided by the singlet Higgs S either real or complex. In this case, the above cancellation between the f_Q and λ_Q couplings does not take place, and hence the scalar FCNC's are still given by Eq. (3.112).

We next consider the basis (b) with $\Delta_{qQ} = \mathbf{0}$. It should be noted that the f_Q and f'_Q couplings may in general take some nonzero values as

$$|(f_Q)_{ia}|, |(f'_Q)_{ia}| \sim \bar{f}_i. \tag{3.115}$$

Then, even though the specific combination $\hat{f}^S_Q$ vanishes due to the condition $\Delta_{qQ} = \mathbf{0}$, there is no reason to have cancellation between the f_Q and f'_Q contributions in Eq. (3.103). Hence, we obtain

$$\Lambda^\alpha_{\mathcal{Q}}[q]_{ij} \sim (m_Q/v_S)\bar{f}_i\epsilon^h_j + (m_{q_i}/v)(\delta_{ij} + \epsilon^h_i\epsilon^h_j). \tag{3.116}$$

The first term from the f_Q and f'_Q couplings is no longer suppressed by the ordinary quark masses. The flavor changing part of the second term from $\Lambda^H_{\mathcal{Q}}[q]$ is related to the Z mediated FCNC's, as given in Eq. (3.111). The λ_Q and λ'_Q contributions in Eq. (3.104) are also estimated as $(m_{q_i}/v_S)\epsilon^h_i\epsilon^h_j$ with Eqs. (3.38) and (3.39). It should here be remembered that in some models the f'_Q coupling is absent. Then, the f_Q coupling is eliminated for $\Delta_{qQ} = \mathbf{0}$, and hence the first term disappears in Eq. (3.116).

The quark mass matrix of the seesaw form may be deformed formally to those in the bases (a) and (b), as seen in Sec. 3.3. Hence, similar features are expected for the neutral scalar couplings, which have been observed so far. For definiteness, we consider the case where the f'_Q coupling is absent and the submatrix M_Q is a bare mass term. Then, the h_q and f_Q contributions are determined, respectively, from Eqs. (3.105) and (3.106) with $\lambda_q = \mathbf{0}$ and $f'_Q = \mathbf{0}$ as

$$\Lambda^\alpha_{\mathcal{Q}}[q]_{ij} \sim (V^\dagger_{q_{\mathrm{R}}}V_{q_{\mathrm{R}}})_{ij}(m_{q_j}/v_S) + (m_{q_i}/v)(V^\dagger_{q_{\mathrm{L}}}V_{q_{\mathrm{L}}})_{ij}. \tag{3.117}$$

Here, the scalar FCNC's ($i \neq j$) are described in terms of the ordinary quark masses and the unitarity violation (3.33) of the ordinary quark mixings $V_{q_{\mathrm{L}}}$ and $V_{q_{\mathrm{R}}}$ induced by the q-Q mixing. In particular, the second term is related to the Z mediated FCNC's. In the case of $m_{Q_1}, m_{Q_2}, m_{Q_3} \sim m_Q$, the first term can be significant with the form as given in Eq. (3.112) for the basis (a). On the other hand, if the inverted hierarchy $m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}$ is realized, these scalar FCNC's are related to the ordinary quark masses with Eqs. (3.57) and (3.58). Hence, they appear to be negligibly small.

FCNC's (Z) versus FCNC's (ϕ)

We have examined so far the structures of the FCNC's for the representative bases, which are summarized in Table 3.2. We here, in particular, note that in some cases the scalar FCNC's (ϕ) can be much larger than the Z mediated FCNC's (Z).

quark mass matrix form $\mathcal{M}_\mathcal{Q}$		gauge-mediated FCNC's ($i \neq j$) $\Delta \mathcal{Z}_\mathcal{Q}[q]_{ij}$	scalar-mediated FCNC's ($i \neq j$) $\Lambda^\alpha_\mathcal{Q}[q]_{ij}$
basis (a) : $\Delta'_{qQ} = \mathbf{0}$		$\frac{m_{q_i}}{m_Q}\frac{m_{q_j}}{m_Q}\epsilon^f_i \epsilon^f_j$	$\frac{m_{q_j}}{v_S}\epsilon^f_i\epsilon^f_j + \frac{m_{q_i}}{v}\Delta\mathcal{Z}_\mathcal{Q}[q]_{ij}$
basis (b) : $\Delta_{qQ} = \mathbf{0}$		$\epsilon^h_i \epsilon^h_j$	$\frac{m_Q}{v_S}\bar{f}_i\epsilon^h_j + \frac{m_{q_i}}{v}\Delta\mathcal{Z}_\mathcal{Q}[q]_{ij}$
seesaw : $M_q = \mathbf{0}$	[com]	$\epsilon^h_i \epsilon^h_j$	$\frac{m_{q_j}}{v_S}\epsilon^f_i\epsilon^f_j + \frac{m_{q_i}}{v}\Delta\mathcal{Z}_\mathcal{Q}[q]_{ij}$
	[inv]	$\frac{m_{q_i}}{\bar{f}v_S}\frac{m_{q_j}}{\bar{f}v_S}$	$\frac{m_{q_i}}{\bar{h}v}\frac{m_{q_j}}{\bar{h}v}\frac{m_{q_j}}{v_S} + \frac{m_{q_i}}{v}\Delta\mathcal{Z}_\mathcal{Q}[q]_{ij}$

Table 3.2: The FCNC's in the gauge and scalar interactions are listed for the representative bases, which are provided by the q-Q mixing. For the seesaw model, [com] indicates the case of comparable singlet quark masses $m_{Q_1}, m_{Q_2}, m_{Q_3} \sim m_Q$, and [inv] the case of inverted hierarchy $m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}$.

In the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, the FCNC's ($\phi$) arise at the first order of q/Q mass ratios, while the FCNC's (Z) are fairly suppressed by the second order of q/Q mass ratios. Hence, as seen in Eqs. (3.112), (3.113) and (3.114), the FCNC's (ϕ) are expected to provide significant physical effects for $m_Q, m_{\phi_\alpha} \sim 100\mathrm{GeV} - 1\mathrm{TeV}$.

In the basis (b) with $\Delta_{qQ} = \mathbf{0}$, as seen in Eq. (3.116), if both the f_Q and f'_Q couplings are present with complex S or several real S's, the FCNC's (ϕ) contains the term which is not related to the FCNC's (Z). Then, the contributions of FCNC's (ϕ) may exceed those of FCNC's (Z) if the f_Q and f'_Q couplings are large enough with $m_Q, m_{\phi_\alpha} \sim 100\mathrm{GeV} - 1\mathrm{TeV}$.

The seesaw model with $m_{Q_1}, m_{Q_2}, m_{Q_3} \sim m_Q$ has a hybrid feature of the above two cases for the FCNC's, i.e., the FCNC's (ϕ) has the structure the same as in the basis (a), while the FCNC's (Z) the same as in the basis (b). In the seesaw model with $m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}$, the FCNC's ($Z$) and FCNC's ($\phi$) are both negligibly small suppressed by the powers of the ordinary quark masses.

We would anyway like to emphasize that in some cases the effects of the FCNC's (ϕ) can be more important than those of the FCNC's (Z). Then, the neutral Higgs contributions to the flavor changing and CP violating processes may rather serve as signals for the new physics beyond the standard model [10, 24, 25, 38]. This possibility has not been paid so much attention before in the models with singlet quarks.

3.5 Numerical analysis

We here perform a detailed numerical analysis for calculating the quark mixings and flavor changing couplings which are induced by the q-Q mixing. The flavor structures of these q-Q mixing effects have been described in the previous sections. They are really confirmed by this numerical analysis.

We begin with taking some reasonable values for the model parameters. The VEV's of the Higgs fields are taken typically as

$$v = 246\mathrm{GeV} \ , \ v_S = 500\mathrm{GeV} \ .$$

The singlet quark masses are chosen as

$$m_{Q_a} \sim 300\mathrm{GeV} - 1\mathrm{TeV} \ (a = 1, 2, \ldots, N_Q)$$

(except for the seesaw model with $m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}$). This is made by taking suitably the λ_Q and λ'_Q couplings with given v_S:

$$|(\lambda_Q)_{ab}|, \ |(\lambda'_Q)_{ab}| \sim \frac{m_Q}{v_S}$$

Then, the f_Q, f'_Q and h_q couplings are taken so as to reproduce the expected values of the q-Q mixing parameters:

$$|(f_Q)_{ia}|, \ |(f'_Q)_{ia}| \to \epsilon^f_i \lesssim 1 \ , \ |(h_q)_{ai}| \to \epsilon^h_i \lesssim 1 \ .$$

The complex phases of these Yukawa couplings and the VEV of the singlet Higgs field are taken randomly in the full range:

$$\arg[h_q, f_Q, f'_Q, \lambda_Q, \lambda'_Q], \ \phi_S \in [-\pi, \pi] \ .$$

The actual masses of the ordinary quarks are reproduced by adjusting the relevant parameters as

$$\text{cases (a) and (b)} : \lambda_{q_i} \to m_{q_i},$$
$$\text{seesaw} : \left\{ \begin{array}{l} (f_Q)_{ia}, (h_q)_{ai} \to m_{q_i} \ (m_{Q_a} \sim m_Q) \ , \\ m_{Q_i} \to m_{q_i} \ (m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}) \ . \end{array} \right.$$

The quark mass matrix $\mathcal{M}_\mathcal{Q}$ given with these parameters is diagonalized numerically. Then, the quark mixing matrices are determined precisely, and the flavor changing couplings are calculated. Some typical results on these q-Q mixing effects are shown in Figs.

3.1 – 3.7 by noting for instance the u-t transition terms. Similar results are also obtained for the other flavor changing terms.

In Fig. 3.1, the u-t mixing elements are shown for the basis (a) with $N_U = 1$ depending on the combination $(\epsilon_1^f \epsilon_3^f)^{1/2}$ of the q-Q mixing parameters. The values of the relevant couplings are taken randomly. The marks are assigned as

$$\begin{array}{lcl} \text{circle} & : & |(V_{u_\mathrm{L}})_{ij}| \ \text{(left-handed)}, \\ \text{triangle} & : & |(V_{u_\mathrm{R}})_{ij}| \ \text{(right-handed)}. \end{array}$$

The respective elements are denoted as

$$\begin{array}{lcl} \text{blank mark} & : & ij = 13, \\ \text{filled mark} & : & ij = 31. \end{array}$$

The dotted lines indicate the expected flavor structures given in Eqs. (3.29) and (3.30).

In Fig. 3.2, the same quantities as in Fig. 3.1 are shown for the basis (b) with $N_U = 3$. The dotted lines indicate the expected flavor structures given in Eqs. (3.40) and (3.41). The duality (3.164) between the bases (a) and (b) is clearly observed in these figures. That is, the regions of the circles (left-handed mixing) and the triangles (right-handed mixing) are exchanged. It is also found that similar q-Q mixing effects are obtained irrespectively of the number N_Q of the singlet quarks.

In Fig. 3.3, the u-t mixing elements are shown for the seesaw models of (i), (ii), (iii) and inverted cases depending on the relevant coupling parameter $\bar{f}_3 \equiv \sum_a |(f_U)_{3a}|/3$. It should here be remarked that $m_t \sim v$ is obtained for $|(f_U)_{3a}| \sim 1$ and $|(h_u)_{a3}| \sim 1$ with $m_Q \sim v_S$. The marks are assigned for the respective cases as

$$\begin{array}{lcl} \text{circle} & : & \text{(i)}, \\ \text{square} & : & \text{(ii)}, \\ \text{triangle} & : & \text{(iii)}, \\ \text{diamond} & : & \text{inverted}. \end{array}$$

The chirality of the mixings is also denoted as

$$\begin{array}{lcl} \text{blank mark} & : & |(V_{u_\mathrm{L}})_{13,31}| \ \text{(left-handed)}, \\ \text{filled mark} & : & |(V_{u_\mathrm{R}})_{13,31}| \ \text{(right-handed)}. \end{array}$$

Here, $|(V_{u_\chi})_{13}|$ and $|(V_{u_\chi})_{31}|$ appear to be of the same order in the seesaw models. The dotted lines indicate the expected values given in Eqs. (3.51), (3.52), (3.53), (3.59) and (3.60). We observe, in particular, that the significant mixings are induced for $(V_{u_\mathrm{L}})_{13}$ (blank square) of case (ii) and $(V_{u_\mathrm{R}})_{31}$ (filled triangle) of case (iii). Although the left-handed mixings $(V_{q_\mathrm{L}})_{ij}$ appear to be of $O(1)$ for the case (ii), its unitarity violation is

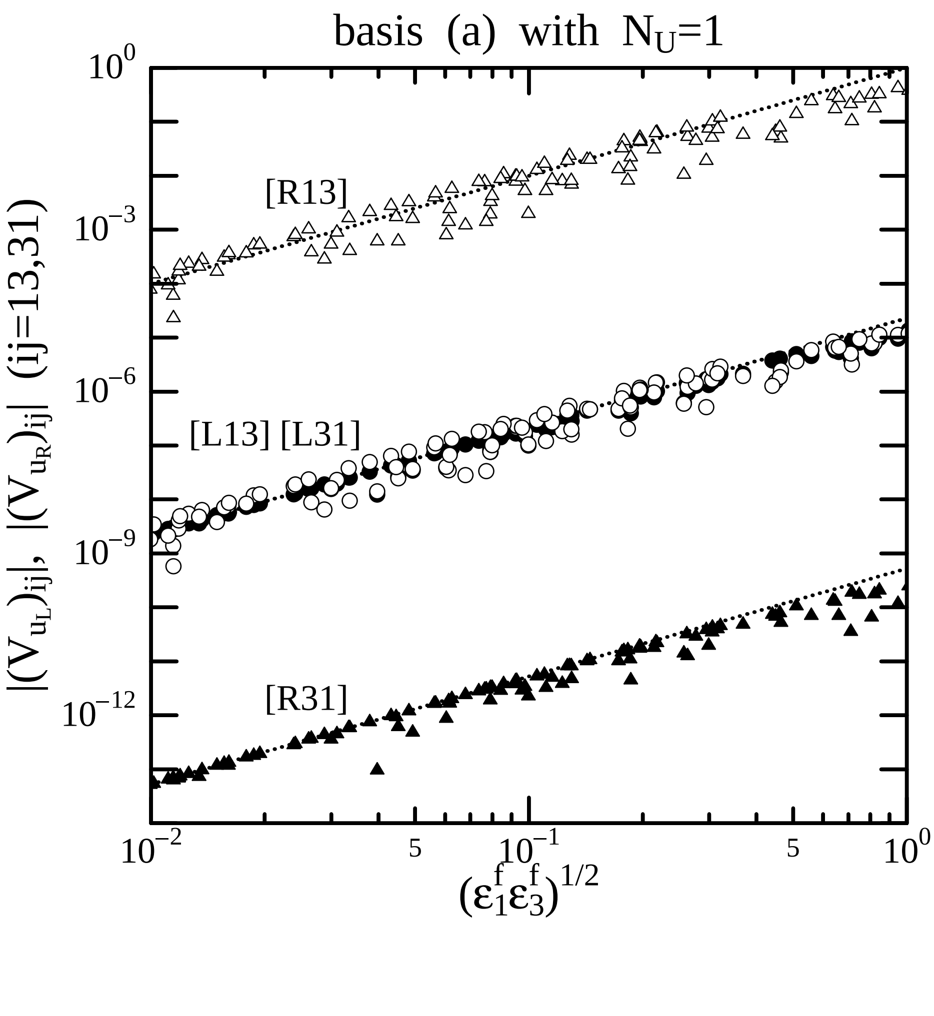

Figure 3.1: The u-t mixing elements are shown for the basis (a) with $N_U = 1$ depending on $(\epsilon_1^f\epsilon_3^f)^{1/2}$. The values of the relevant couplings are taken randomly. The marks are assigned as "circle" : $|(V_{u_\mathrm{L}})_{ij}|$ and "triangle" : $|(V_{u_\mathrm{R}})_{ij}|$. The respective elements are denoted as "blank mark" : $ij = 13$ and "filled mark" : $ij = 31$. The dotted lines indicate the expected flavor structures.

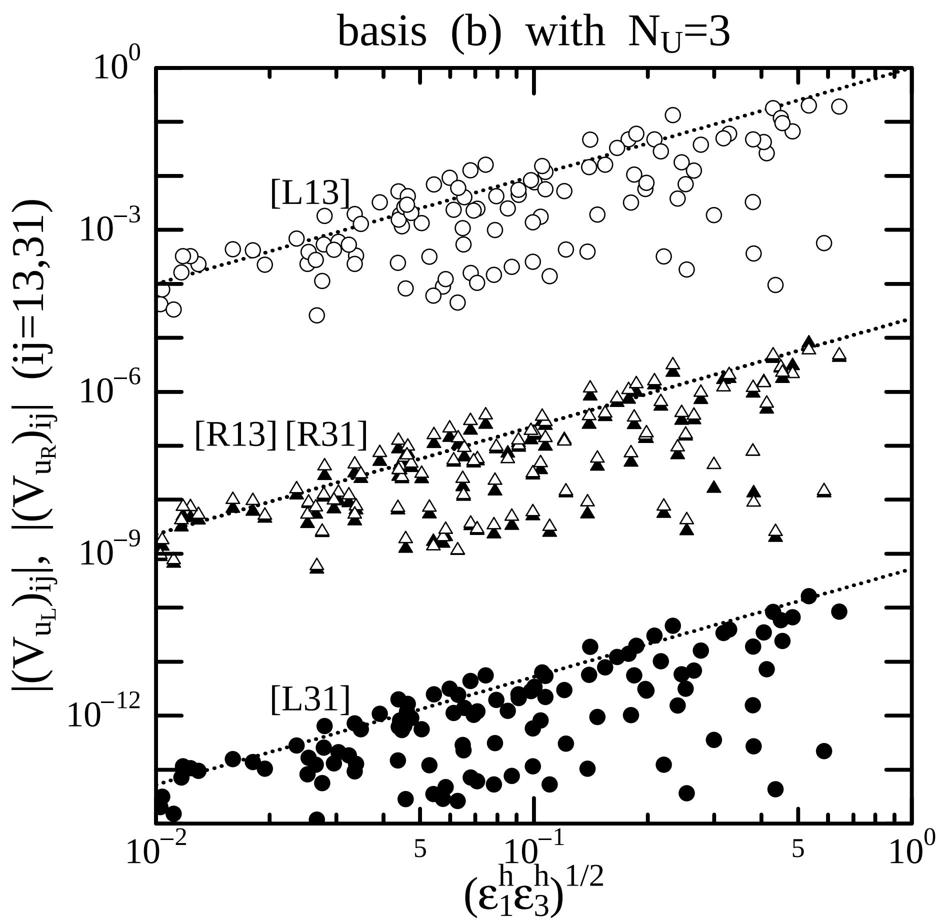

Figure 3.2: The u-t mixing elements are shown for the basis (b) with $N_U = 3$ depending on $(\epsilon_1^h \epsilon_3^h)^{1/2}$. The marks are assigned the same as in Fig. 3.1. The dotted lines indicate the expected flavor structures.

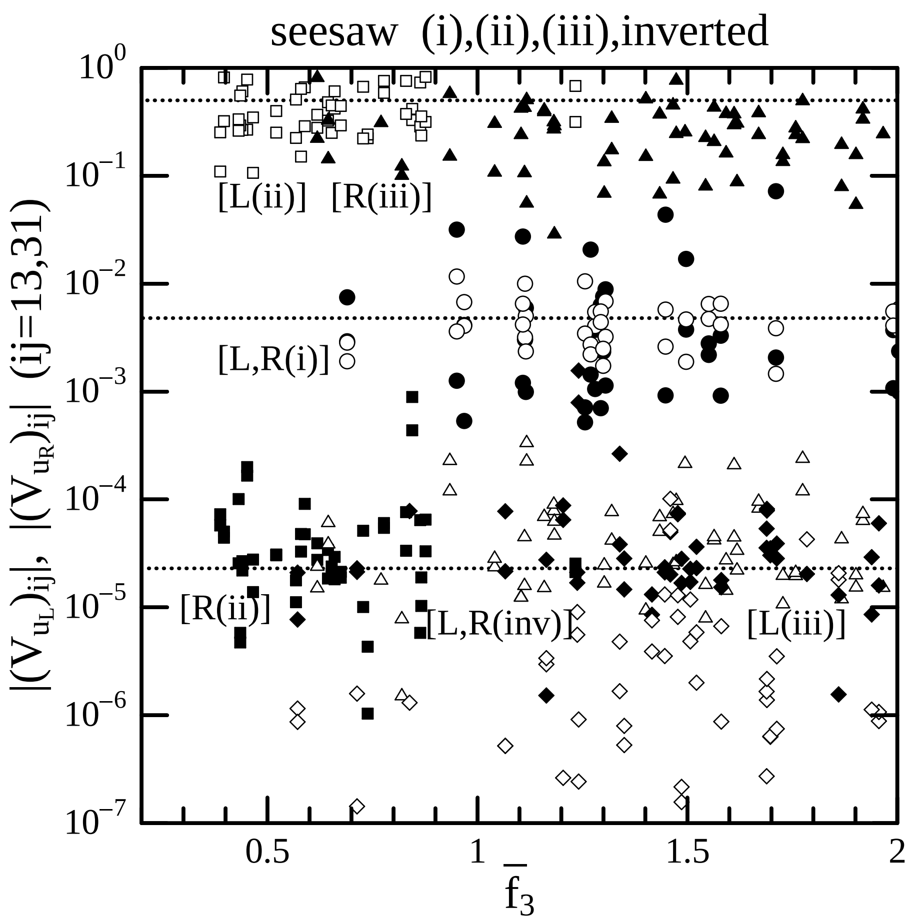

Figure 3.3: The u-t mixing elements are shown depending on $\bar{f}_3$ for the seesaw models of (i), (ii), (iii) and inverted cases. The marks are assigned as "circle" : (i), "square" : (ii), "triangle" : (iii) and "diamond" : inverted. The chirality of the mixings is also denoted as "blank mark" : $|(V_{u_{\rm L}})_{13,31}|$ and "filled mark" : $|(V_{u_{\rm R}})_{13,31}|$. The dotted lines indicate the expected values.

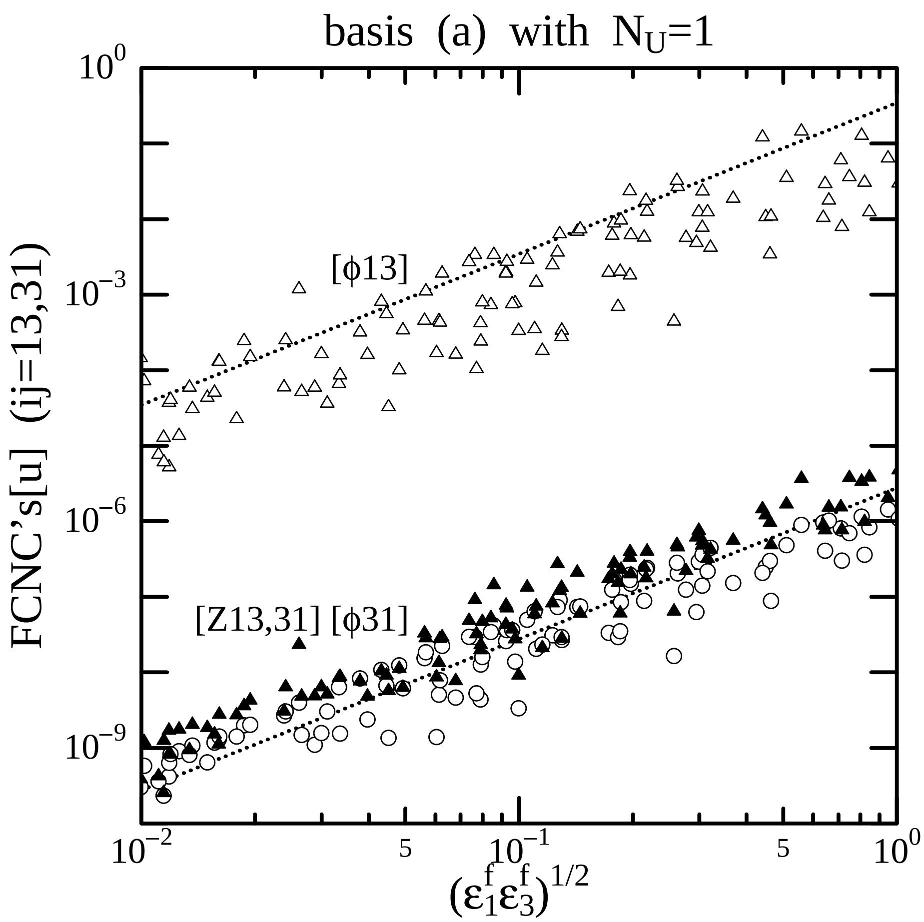

Figure 3.4: The FCNC's of the u-t transition are shown for the basis (a) with $N_U = 1$ depending on $(\epsilon_1^f \epsilon_3^f)^{1/2}$. The marks are assigned as "circle (blank)": $|\Delta \mathcal{Z}_\mathcal{U}[u]_{13}| = |\Delta \mathcal{Z}_\mathcal{U}[u]_{31}|$, "triangle (blank)" : $\bar{\Lambda}_\mathcal{U}[u]_{13}$ and "triangle (filled)" : $\bar{\Lambda}_\mathcal{U}[u]_{31}$. The dotted lines indicate the expected flavor structures.

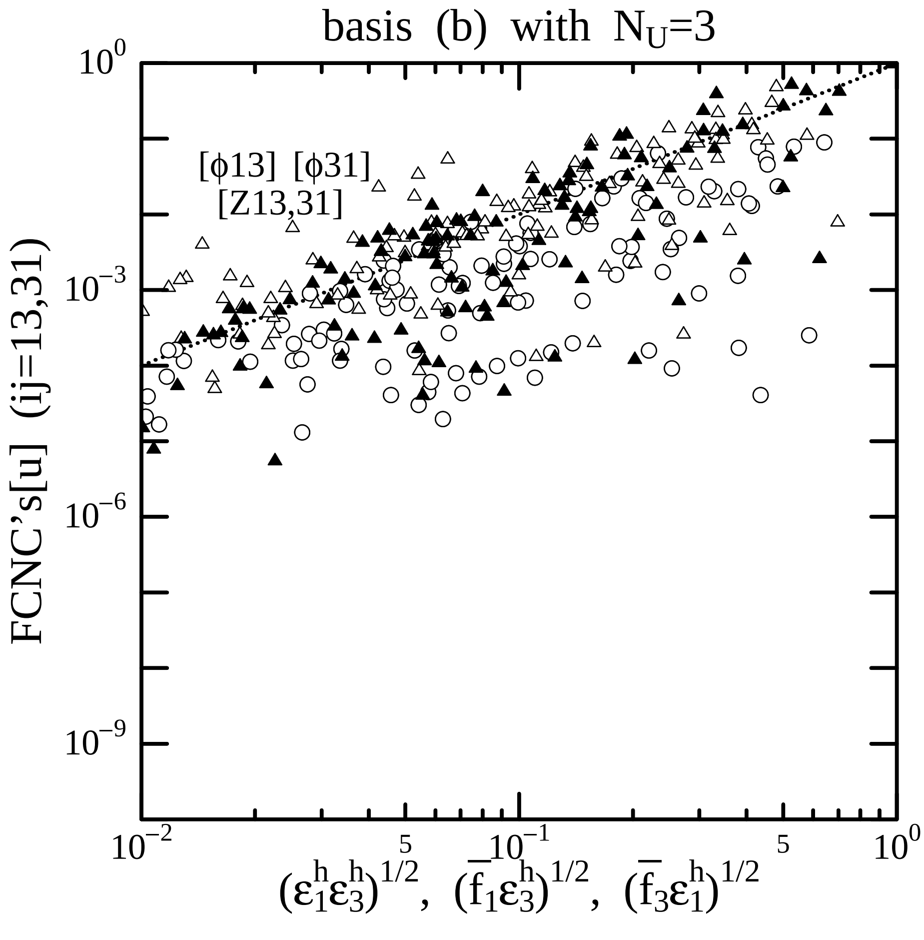

Figure 3.5: The FCNC's of the u-t transition are shown for the basis (b) with $N_U = 3$. They depend, respectively, on $(\epsilon_1^h \epsilon_3^h)^{1/2}$ for the gauge couplings and $(\bar{f}_i \epsilon_j^h)^{1/2}$ $(ij = 13, 31)$ for the scalar couplings. The marks are assigned the same as in Fig. 3.4. The dotted lines indicate the expected flavor structures.

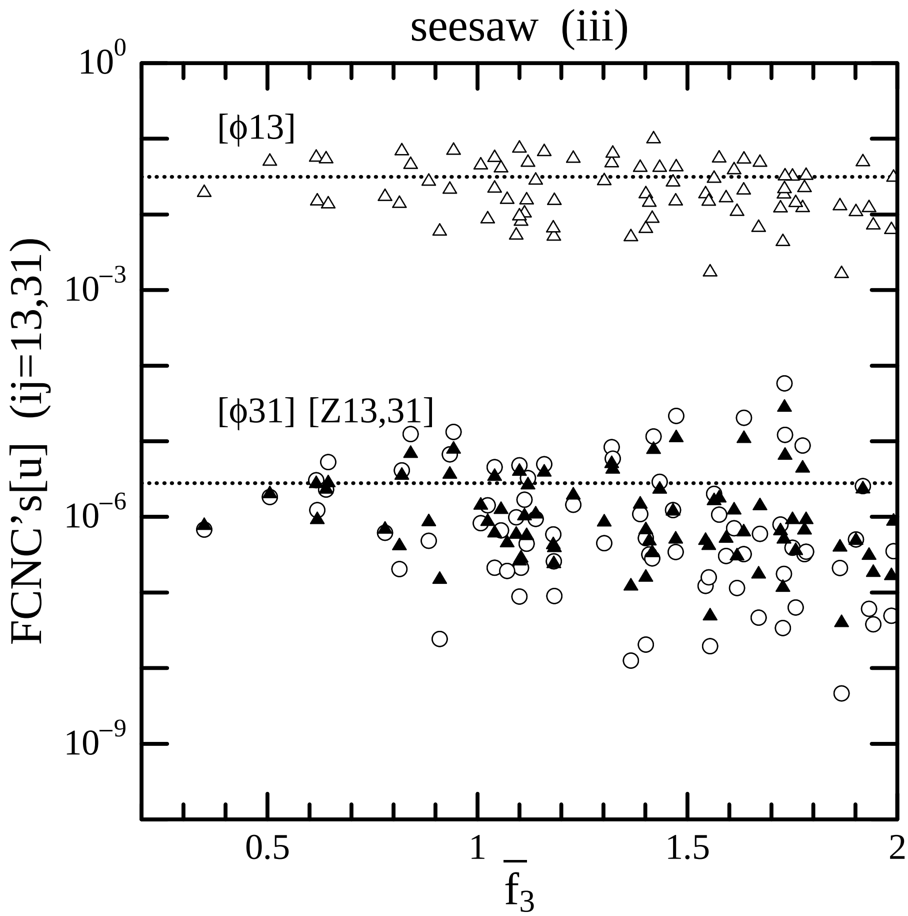

Figure 3.6: The FCNC's of the u-t transition are shown for the seesaw model of (iii) depending on $\bar{f}_3$. The dotted lines indicate the expected values.

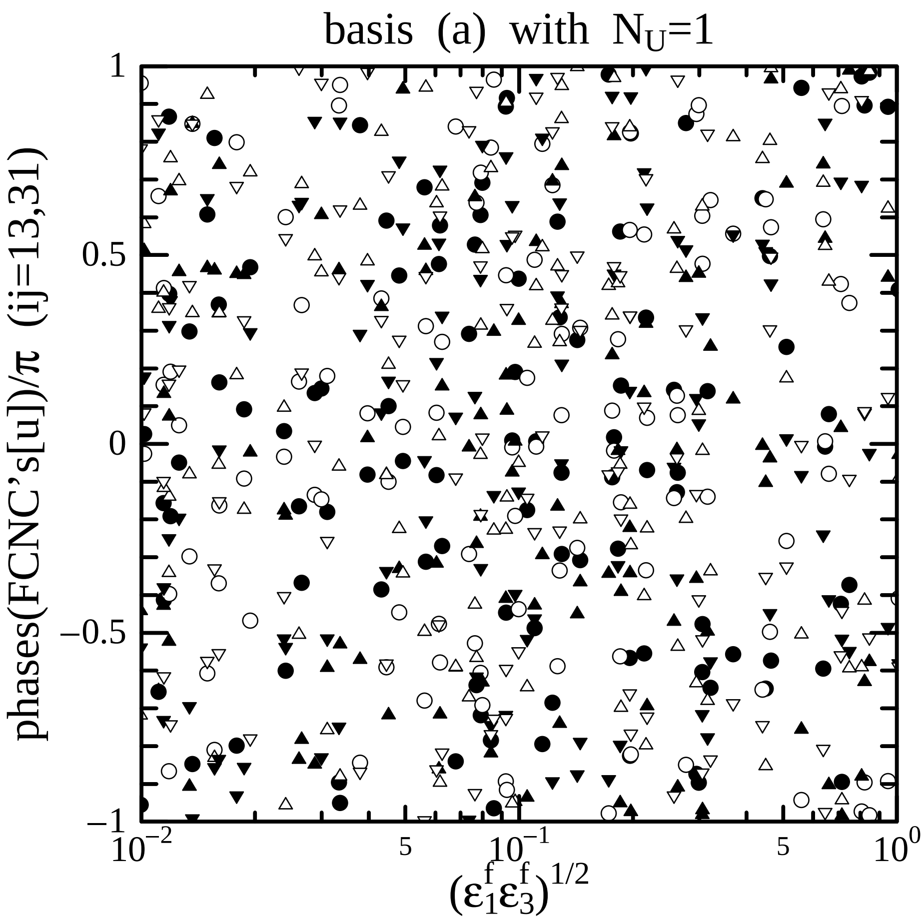

Figure 3.7: The complex phases involved in the FCNC's of the u-t transition are shown for the basis (a) with $N_U = 1$. The marks are assigned as "circle" : $\arg[\Delta\mathcal{Z}_{\mathcal{U}}[u]_{ij}]$, "triangle-up" : $\arg[\Lambda^{S_+}_{\mathcal{U}}[u]_{ij}]$ and "triangle-down" : $\arg[\Lambda^{S_-}_{\mathcal{U}}[u]_{ij}]$. The respective elements are denoted as "blank mark" : $ij = 13$ and "filled mark" : $ij = 31$.

suppressed by $\epsilon_i^h \epsilon_j^h \sim (\bar{h}v/m_Q)^2$. Hence, in this specific case with $m_t \sim \bar{h}v$, the singlet quark masses $\sim m_U$ will be required to be sufficiently larger than the electroweak scale [9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. The right-handed mixings $(V_{q_\mathrm{R}})_{ij} \sim 1$ for the case (iii), on the other hand, contribute to provide the significant scalar couplings $\Lambda_{\mathcal{Q}}^\alpha[q]_{ij} \sim (m_{q_j}/v_S)(\bar{f}_3 v_S/m_Q)^2$.

The magnitudes of the FCNC's of the u-t transition are shown in Figs. 3.4, 3.5 and 3.6, respectively, for the basis (a), basis (b) and seesaw of case (iii). In these cases, the significant FCNC's (ϕ) are obtained, which are not related to the FCNC's (Z). The marks are assigned as

$$\begin{array}{lcl} \text{circle (blank)} & : & |\Delta\mathcal{Z}_{\mathcal{U}}[u]_{13}| = |\Delta\mathcal{Z}_{\mathcal{U}}[u]_{31}|, \\ \text{triangle (blank)} & : & \bar{\Lambda}_{\mathcal{U}}[u]_{13}, \\ \text{triangle (filled)} & : & \bar{\Lambda}_{\mathcal{U}}[u]_{31}, \end{array}$$

where

$$\bar{\Lambda}_{\mathcal{Q}}[q]_{ij} \equiv \frac{1}{3}\left[|\Lambda_{\mathcal{Q}}^H[q]_{ij}| + |\Lambda_{\mathcal{Q}}^{S_+}[q]_{ij}| + |\Lambda_{\mathcal{Q}}^{S_-}[q]_{ij}| \right].$$

These FCNC's are shown depending on the relevant parameters ϵ_i^f, ϵ_i^h and $\bar{f}_i \equiv \sum_a |(f_U)_{ia}|/3$ ($|(f_U)_{ia}| = |(f_U')_{ia}|$ in the basis (b) with $\Delta_{qQ}' = \mathbf{0}$), so that their flavor structures are readily compared to the expected ones (dotted lines). We observe clearly that in the basis (a) and seesaw of case (iii) the scalar coupling (13 element) of the u-t transition is quite significant being proportional to the top quark mass m_t. On the other hand, in the basis (b) the gauge couplings as well as the scalar couplings can be considerable.

In Fig. 3.7, the complex phases involved in the FCNC's of the u-t transition are shown for the basis (a). The marks (blank for $ij = 13$ and filled for $ij = 31$) are assigned as

$$\begin{array}{lcl} \text{circle} & : & \arg[\Delta\mathcal{Z}_{\mathcal{U}}[u]_{ij}], \\ \text{triangle-up} & : & \arg[\Lambda_{\mathcal{U}}^{S_+}[u]_{ij}], \\ \text{triangle-down} & : & \arg[\Lambda_{\mathcal{U}}^{S_-}[u]_{ij}]. \end{array}$$

[Note that $\arg[\Lambda_{\mathcal{Q}}^H[q]_{ij}] = \arg[\Delta\mathcal{Z}_{\mathcal{Q}}[q]_{ij}]$ due to Eq. (3.111).] Here, the complex phases of the relevant parameters are taken randomly. It is clearly observed that the CP violating phases in the gauge and scalar couplings take various values depending on the phases of the original model parameters.

3.6 Summary and discussion

The singlet quarks may provide various intriguing effects in particle physics and cosmology through the mixing with the ordinary quarks. We have presented the systematic and comprehensive investigations on the quark mixings in the electroweak models with singlet quarks. There are some appropriate choices of the quark basis for the electroweak eigenstates, where the entire quark mass matrix $\mathcal{M}_Q$ has the specific form without loss of generality. They are the basis (a) with the $Q^c q$ mixing term $\Delta'_{qQ} = \mathbf{0}$, the basis (b) with the $q^c Q$ mixing term $\Delta_{qQ} = \mathbf{0}$ and the seesaw with the ordinary quark mass matrix $M_q = \mathbf{0}$. We may take complementarily these quark bases, depending on the model parameter ranges and also the symmetries and matter contents.

We have examined in detail for these bases how the ordinary quark masses and mixings are affected by the q-Q mixing. The flavor changing interactions are also modified by the q-Q mixing. Specifically, the CKM unitarity within the ordinary quark sector is violated, and the FCNC's arise both in the gauge and scalar couplings. The structures of these flavor changing interactions have been clarified for the respective quark bases. In fact, they are described appropriately in terms of the q-Q mixing parameters and the quark masses. These results ensure that there are some reasonable ranges of the model parameters where the ordinary quark mass hierarchy and the actual CKM structure are reproduced even in the presence of singlet quarks. In these meanings, the present case with singlet quarks may provide an interesting extension of the idea of natural flavor conservation [29, 30].

A detailed numerical analysis has further been performed for calculating precisely the quark mixings and flavor changing couplings with singlet quarks. Then, it has been confirmed that the q-Q mixing effects really exhibit the expected flavor structures. These calculations on the singlet quarks may be extended readily for the models with various exotic quarks and leptons such as vector-like electroweak doubles, where the entire fermion mass matrix has the same form as $\mathcal{M}_Q$.

We finally discuss some phenomenological implications derived from the results of the present investigations. We particularly note that the scalar FCNC's (ϕ) are sometimes fairly larger than the gauge FCNC's (Z), as seen in Figs. 3.4, 3.5 and 3.6. Then, if the singlet quarks and extra Higgs particles exist just above the electroweak scale with $m_Q, m_{\phi_\alpha} \sim 100\mathrm{GeV} - 1\mathrm{TeV}$, the scalar FCNC's ($\phi$) are expected to provide significant effects on the flavor changing and CP violating processes, e.g., the K, B and D meson physics. Hence, these effects of scalar FCNC's (ϕ) rather than those of gauge FCNC's (Z) might serve as signals for the new physics beyond the standard model. This possibility has not been paid so much attention before in the models with singlet quarks.

The FCNC's induced by the q-Q mixing may sometimes be related to the ordinary quark masses. Then, it will be expected that the q-Q mixing effects can be observed most likely in the top quark physics. In particular, in the basis (a) the scalar FCNC's (ϕ) and the gauge FCNC's (Z) arise, respectively, at the first and second orders of the ordinary quark masses. Then, we expect $\mathrm{Br}(t \to cZ) \sim 10^{-5}$ with $|(\Delta \mathcal{Z}_{\mathcal{U}})_{13}| \sim (m_c/m_Q)(m_t/m_Q)$ for $m_Q \sim 200\mathrm{GeV}$ and $\epsilon_i^f \sim 1$ while negligibly small $\mathrm{Br}(t \to uZ)$. If the neutral Higgs particle ϕ_1, which is mainly h_1 of the standard model, is light enough, we would obtain the top quark decays involving ϕ_1 [61]. The branching ratio can be rather significant with the scalar couplings $|(\Lambda_{\mathcal{Q}}^{S_\pm})_{i3}| \sim (m_t/v_S)\epsilon_i^f \epsilon_3^f$ and the H-S mixings $O_{21}, O_{31} \sim v/v_S$. In fact, we estimate $\mathrm{Br}(t \to q_i + \phi_1) \sim 10^{-2}(\epsilon_i^f \epsilon_3^f)^2$ ($q_i = u, c$) for $v_S \sim 500\mathrm{GeV}$ and $m_{\phi_1} \simeq 100\mathrm{GeV}$.

It will be worth making further investigations of these remarkable effects particularly of the scalar FCNC's induced by the q-Q mixing.

Appendix A
Diagonalization of the quark mass matrix

We here present the algebraic calculations for diagonalizing the quark mass matrix $\mathcal{M}_{\mathcal{Q}}$. The leading order results for $N_Q = 1$ are given in the literature [5, 6, 7, 8]. We do not intend to calculate explicitly the higher order corrections. The following treatments hence seems to be of little practical use. The diagonalization of $\mathcal{M}_{\mathcal{Q}}$ can anyway be made precisely by numerical calculations. We would rather like to present comprehensive explanations for the specific flavor structures of the q-Q mixing effects. They are even valid beyond the leading orders for more general cases with several singlet quarks and wide ranges of the model parameters. Then, the following arguments appear to be helpful to understand the results of precise numerical calculations.

The diagonalization of $\mathcal{M}_{\mathcal{Q}}$ may be performed at two steps by dividing the unitary transformation as

$$\mathcal{V}_{\mathcal{Q}_\chi} = \mathcal{V}_{\mathcal{Q}_\chi}^{(1)} \mathcal{V}_{\mathcal{Q}_\chi}^{(2)} \tag{3.118}$$

with

$$\mathcal{V}_{\mathcal{Q}_\chi}^{(1)} = \begin{pmatrix} V_{q_\chi}^{(1)} & \epsilon_{q_\chi}^{(1)} \\ -\epsilon_{q_\chi}^{(1)\dagger} & V_{Q_\chi}^{(1)} \end{pmatrix}, \tag{3.119}$$

$$\mathcal{V}_{\mathcal{Q}_\chi}^{(2)} = \begin{pmatrix} V_{q_\chi}^{(2)} & \mathbf{0} \\ \mathbf{0} & V_{Q_\chi}^{(2)} \end{pmatrix}. \tag{3.120}$$

Here, $V_{q_\chi}^{(1)}$ and $V_{Q_\chi}^{(1)}$ are in general non-unitary due to the q-Q mixing $\epsilon_{q_\chi}^{(1)}$ ($\epsilon_{q_\chi}^{(1)\prime} = \epsilon_{q_\chi}^{(1)}$, as given explicitly below), while $V_{q_\chi}^{(2)}$ and $V_{Q_\chi}^{(2)}$ are unitary by definition with $\epsilon_{q_\chi}^{(2)} = \epsilon_{q_\chi}^{(2)\prime} = \mathbf{0}$. The components of the entire transformation $\mathcal{V}_{\mathcal{Q}_\chi}$ are then given by

$$
\begin{aligned}
\epsilon_{q_\chi} &= \epsilon_{q_\chi}^{(1)} V_{Q_\chi}^{(2)}, \ \epsilon'_{q_\chi} = V_{q_\chi}^{(2)\dagger} \epsilon_{q_\chi}^{(1)}, && (3.121)\\
V_{q_\chi} &= V_{q_\chi}^{(1)} V_{q_\chi}^{(2)}, \ V_{Q_\chi} = V_{Q_\chi}^{(1)} V_{Q_\chi}^{(2)}. && (3.122)
\end{aligned}
$$

The first step transformation is utilized for eliminating Δ_{qQ} and Δ'_{qQ}, which may be given by

$$
\mathcal{V}_{\mathcal{Q}_\chi}^{(1)} = \exp[\mathcal{E}_{\mathcal{Q}_\chi}] = \mathbf{1} + \mathcal{E}_{\mathcal{Q}_\chi} + \frac{1}{2!}\mathcal{E}_{\mathcal{Q}_\chi}^2 + \cdots \qquad (3.123)
$$

with certain anti-hermitian matrix

$$
\mathcal{E}_{\mathcal{Q}_\chi} = \begin{pmatrix} \mathbf{0} & \bar{\epsilon}_{q_\chi} \\ -\bar{\epsilon}_{q_\chi}^\dagger & \mathbf{0} \end{pmatrix}. \qquad (3.124)
$$

Then, we have the submatrices in $\mathcal{V}_{\mathcal{Q}_\chi}^{(1)}$ as

$$
\begin{aligned}
\epsilon_{q_\chi}^{(1)} &= \epsilon_{q_\chi}^{(1)\prime} = \bar{\epsilon}_{q_\chi} - \frac{1}{3!}\bar{\epsilon}_{q_\chi}\bar{\epsilon}_{q_\chi}^\dagger \bar{\epsilon}_{q_\chi} + \dots \\
&= \bar{\epsilon}_{q_\chi} \bar{B}_{q_\chi} = \bar{B}'_{q_\chi} \bar{\epsilon}_{q_\chi}, && (3.125)\\
V_{q_\chi}^{(1)} &= V_{q_\chi}^{(1)\dagger} = \mathbf{1} - \frac{1}{2!}\bar{\epsilon}_{q_\chi}\bar{\epsilon}_{q_\chi}^\dagger + \cdots \\
&= \mathbf{1} + \bar{\epsilon}_{q_\chi} \bar{A}_{q_\chi} \bar{\epsilon}_{q_\chi}^\dagger = \mathbf{1} + \epsilon_{q_\chi}^{(1)} A_{q_\chi} \epsilon_{q_\chi}^{(1)\dagger}, && (3.126)\\
V_{Q_\chi}^{(1)} &= V_{Q_\chi}^{(1)\dagger} = \mathbf{1} - \frac{1}{2!}\bar{\epsilon}_{q_\chi}^\dagger \bar{\epsilon}_{q_\chi} + \cdots \\
&= \mathbf{1} + \bar{\epsilon}_{q_\chi}^\dagger \bar{A}_{Q_\chi} \bar{\epsilon}_{q_\chi} = \mathbf{1} + \epsilon_{q_\chi}^{(1)\dagger} A_{Q_\chi} \epsilon_{q_\chi}^{(1)}, && (3.127)
\end{aligned}
$$

where $\bar{A}_{q_\chi}$, $\bar{A}_{Q_\chi}$, $\bar{B}_{q_\chi}$, $\bar{B}'_{q_\chi} = \mathbf{1} + O(\bar{\epsilon}_{q_\chi}^2)$, and $A_{q_\chi} = \bar{B}_{q_\chi}^{-1} \bar{A}_{q_\chi} \bar{B}_{q_\chi}^{\dagger -1}$, $A_{Q_\chi} = \bar{B}_{q_\chi}^{\prime\dagger -1} \bar{A}_{Q_\chi} \bar{B}_{q_\chi}^{\prime -1}$. The quark mass matrix is transformed as

$$
\mathcal{V}_{\mathcal{Q}_\mathrm{R}}^{(1)\dagger} \mathcal{M}_Q \mathcal{V}_{\mathcal{Q}_\mathrm{L}}^{(1)} = \begin{pmatrix} M_q^{(1)} & \mathbf{0} \\ \mathbf{0} & M_Q^{(1)} \end{pmatrix}. \qquad (3.128)
$$

Here, the first step transformation is determined by the conditions,

$$
\begin{aligned}
\Delta_{qQ}^{(1)} &= V_{q_\mathrm{R}}^{(1)\dagger}(M_q \epsilon_{q_\mathrm{L}}^{(1)} + \Delta_{qQ} V_{Q_\mathrm{L}}^{(1)}) - \epsilon_{q_\mathrm{R}}^{(1)}(M_Q V_{Q_\mathrm{L}}^{(1)} + \Delta'_{qQ} \epsilon_{q_\mathrm{L}}^{(1)}) = \mathbf{0}, && (3.129)\\
\Delta_{qQ}^{(1)\prime} &= (V_{Q_\mathrm{R}}^{(1)\dagger} \Delta'_{qQ} + \epsilon_{q_\mathrm{R}}^{(1)\dagger} M_q) V_{q_\mathrm{L}}^{(1)} - (V_{Q_\mathrm{R}}^{(1)\dagger} M_Q + \epsilon_{q_\mathrm{R}}^{(1)\dagger} \Delta_{qQ}) \epsilon_{q_\mathrm{L}}^{(1)\dagger} = \mathbf{0}. && (3.130)
\end{aligned}
$$

These $2 \times 3 \times N_Q$ conditions are just satisfied by the $2 \times 3 \times N_Q$ parameters contained in $\bar{\epsilon}_{q_\mathrm{L}}$ and $\bar{\epsilon}_{q_\mathrm{R}}$. Practically, in these conditions (3.129) and (3.130) the ordinary quark mixing matrices $V^{(1)}_{q_\chi}$ may be expressed in terms of the q-Q mixing matrices $\epsilon^{(1)}_{q_\chi}$ with Eq. (3.126). Then, by performing some algebra we obtain the relations,

$$\begin{aligned} \epsilon^{(1)}_{q_{\rm L}} &= (\Delta'^\dagger_{qQ} + M^\dagger_q \epsilon^{(1)}_{q_{\rm R}} V^{(1)-1}_{Q_{\rm R}})(M^\dagger_Q + \delta^\dagger_{Q_{\rm L}})^{-1}, && (3.131)\\ \epsilon^{(1)}_{q_{\rm R}} &= (\Delta_{qQ} + M_q \epsilon^{(1)}_{q_{\rm L}} V^{(1)-1}_{Q_{\rm L}})(M_Q + \delta_{Q_{\rm R}})^{-1}, && (3.132) \end{aligned}$$

where

$$\begin{aligned} \delta_{Q_{\rm L}} &= V^{(1)\dagger-1}_{Q_{\rm R}} \epsilon^{(1)\dagger}_{q_{\rm R}} \Delta_{qQ} - \Delta'_{qQ} \epsilon^{(1)}_{q_{\rm L}} A^\dagger_{q_{\rm L}} - V^{(1)\dagger-1}_{Q_{\rm R}} \epsilon^{(1)\dagger}_{q_{\rm R}} M_q \epsilon^{(1)}_{q_{\rm L}} A^\dagger_{q_{\rm L}}, && (3.133)\\ \delta_{Q_{\rm R}} &= -A_{q_{\rm R}} \epsilon^{(1)\dagger}_{q_{\rm R}} \Delta_{qQ} + \Delta'^\dagger_{qQ} \epsilon^{(1)}_{q_{\rm L}} V^{(1)-1}_{Q_{\rm L}} - A_{q_{\rm R}} \epsilon^{(1)\dagger}_{q_{\rm R}} M_q \epsilon^{(1)}_{q_{\rm L}} V^{(1)-1}_{Q_{\rm L}}. && (3.134) \end{aligned}$$

The effective quark mass matrices obtained at the first step are given by

$$\begin{aligned} M^{(1)}_q &= (V^{(1)\dagger}_{q_{\rm R}} M_q - \epsilon^{(1)}_{q_{\rm R}} \Delta'_{qQ}) V^{(1)}_{q_{\rm L}} - (V^{(1)\dagger}_{q_{\rm R}} \Delta_{qQ} - \epsilon^{(1)}_{q_{\rm R}} M_Q) \epsilon^{(1)\dagger}_{q_{\rm L}}, && (3.135)\\ M^{(1)}_Q &= (V^{(1)\dagger}_{Q_{\rm R}} M_Q + \epsilon^{(1)\dagger}_{q_{\rm R}} \Delta_{qQ}) V^{(1)}_{Q_{\rm L}} + (\epsilon^{(1)\dagger}_{q_{\rm R}} M_q + V^{(1)\dagger}_{Q_{\rm R}} \Delta'_{qQ}) \epsilon^{(1)}_{q_{\rm L}}. && (3.136) \end{aligned}$$

Here, $M^{(1)}_q$ for the ordinary quarks is, in particular, calculated by considering the relation $-(V^{(1)\dagger}_{q_{\rm R}} \Delta_{qQ} - \epsilon^{(1)}_{q_{\rm R}} M_Q) = (V^{(1)\dagger}_{q_{\rm R}} M_q - \epsilon^{(1)}_{q_{\rm R}} \Delta'_{qQ}) \epsilon^{(1)}_{q_{\rm L}} V^{(1)-1}_{Q_{\rm L}}$ from Eq. (3.129) as

$$M^{(1)}_q = (V^{(1)\dagger}_{q_{\rm R}} M_q - \epsilon^{(1)}_{q_{\rm R}} \Delta'_{qQ})(\mathbf{1} + R_q) \ , \tag{3.137}$$

where

$$R_q = \epsilon^{(1)}_{q_{\rm L}} (A_{q_{\rm L}} + V^{(1)-1}_{Q_{\rm L}}) \epsilon^{(1)\dagger}_{q_{\rm L}}. \tag{3.138}$$

The effective mass matrices $M^{(1)}_q$ and $M^{(1)}_Q$ at the first step are generally non-diagonal. They are diagonalized at the second step as

$$\begin{aligned} V^{(2)\dagger}_{q_{\rm R}} M^{(1)}_q V^{(2)}_{q_{\rm L}} &= \bar{M}_q, && (3.139)\\ V^{(2)\dagger}_{Q_{\rm R}} M^{(1)}_Q V^{(2)}_{Q_{\rm L}} &= \bar{M}_Q. && (3.140) \end{aligned}$$

This completes the diagonalization of $\mathcal{M}_\mathcal{Q}$.

Basis (a) with $\Delta'_{qQ} = \mathbf{0}$

In the basis (a) with $\Delta'_{qQ} = \mathbf{0}$, the q-Q mixing is generated by the Δ_{qQ} term. Then, the q-Q mixing matrices at the first step are determined with Eqs. (3.131) and (3.132) as

$$\begin{aligned} (\epsilon^{(1)}_{q_{\rm L}})_{ia} &\sim (m_{q_i}/m_Q)\epsilon^f_i, && (3.141)\\ (\epsilon^{(1)}_{q_{\rm R}})_{ia} &\sim \epsilon^f_i, && (3.142) \end{aligned}$$

where the relation $m^0_{q_i} \sim m_{q_i}$, as seen later in Eq. (3.155), is considered for $(M_q)_{ij} = m^0_{q_i}\delta_{ij}$. We also obtain form Eq. (3.126) with Eqs. (3.141) and (3.142)

$$(V^{(1)}_{q_{\rm L}})_{ij} \sim \delta_{ij} + (m_{q_i}/m_Q)(m_{q_j}/m_Q)\epsilon^f_i\epsilon^f_j, \tag{3.143}$$

$$(V^{(1)}_{q_{\rm R}})_{ij} \sim \delta_{ij} + \epsilon^f_i\epsilon^f_j. \tag{3.144}$$

The effective mass matrix $M^{(1)}_q$ for the ordinary quarks is given from Eq. (3.137) with $\Delta'_{qQ} = \mathbf{0}$ as

$$M^{(1)}_q = V^{(1)\dagger}_{q_{\rm R}} M_q(\mathbf{1} + R_q). \tag{3.145}$$

The structure of R_q is specified in Eq. (3.138) with Eq. (3.141) as

$$(R_q)_{ij} \sim (m_{q_i}/m_Q)(m_{q_j}/m_Q)\epsilon^f_i\epsilon^f_j. \tag{3.146}$$

It is suitable to modify $M^{(1)}_q$ as

$$M^{(1')}_q = V^{(2')\dagger}_{q_{\rm R}} M^{(1)}_q = V^{(1')\dagger}_{q_{\rm R}} M_q(\mathbf{1} + R_q) \tag{3.147}$$

by deforming $V^{(1)}_{q_{\rm R}}$ into a triangular form

$$V^{(1')}_{q_{\rm R}} = V^{(1)}_{q_{\rm R}} V^{(2')}_{q_{\rm R}} \sim \begin{pmatrix} 1 & \epsilon^f_1\epsilon^f_2 & \epsilon^f_1\epsilon^f_3 \\ 0 & 1 & \epsilon^f_2\epsilon^f_3 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.148}$$

Here, the relevant unitary transformation is given as

$$(V^{(2')}_{q_{\rm R}})_{ij} \sim \delta_{ij} + \epsilon^f_i\epsilon^f_j. \tag{3.149}$$

Then, by considering the hierarchy $m_{q_1} \ll m_{q_2} \ll m_{q_3} \lesssim m_Q$, we obtain the relations,

$$(M^{(1')\dagger}_q M^{(1')}_q)_{ij} \sim \delta_{ij} m^2_{q_i} + m_{q_i} m_{q_j}\epsilon^f_i\epsilon^f_j, \tag{3.150}$$

$$(M^{(1')}_q M^{(1')\dagger}_q)_{ij} \sim \delta_{ij} m^2_{q_i} + \theta^k_{ij} m^2_{q_k}\epsilon^f_i\epsilon^f_j, \tag{3.151}$$

where

$$\theta^k_{ij} = \begin{cases} 1 & (k = i \text{ for } i < j,\ k = j \text{ for } i > j) \\ 0 & (\text{otherwise}) \end{cases}.$$

The effective mass matrix $M^{(1')}_q$ and its squared ones (3.150) and (3.151) are diagonalized by the unitary transformations,

$$(V^{(2)}_{q_{\rm L}})_{ij} \sim \delta_{ij} + \frac{m_{q_i} m_{q_j}}{m^2_{q_i} + m^2_{q_j}}\epsilon^f_i\epsilon^f_j, \tag{3.152}$$

$$(V^{(2'')}_{q_{\rm R}})_{ij} \sim \delta_{ij} + \frac{m^2_{q_j}}{m^2_{q_i} + m^2_{q_j}}\epsilon^f_i\epsilon^f_j. \tag{3.153}$$

The second step transformation of the right-handed ordinary quarks is then given by

$$V_{q_{\mathrm{R}}}^{(2)} = V_{q_{\mathrm{R}}}^{(2')} V_{q_{\mathrm{R}}}^{(2'')}. \tag{3.154}$$

The ordinary quark masses are obtained as

$$m_{q_i} = (\lambda_{q_i} v/\sqrt{2}) \left[1 + \xi_{q_i}(\boldsymbol{\epsilon}^f)(\epsilon_i^f)^2 \right] \tag{3.155}$$

with certain factors $\xi_{q_i}(\boldsymbol{\epsilon}^f) \sim 1$, which are in fact $-1/2$ in the leading order for $N_Q = 1$.

The net effects on the quark mixings involving the ordinary quarks are calculated in Eqs. (3.121) and (3.122) as

$$\epsilon_{q_\chi},\ \epsilon'_{q_\chi} \sim \epsilon_{q_\chi}^{(1)}\ (\chi = \mathrm{L}, \mathrm{R}), \tag{3.156}$$

$$V_{q_{\mathrm{L}}} \sim V_{q_{\mathrm{L}}}^{(2)},\ V_{q_{\mathrm{R}}} \sim V_{q_{\mathrm{R}}}^{(2'')}. \tag{3.157}$$

The symbol "$\sim$" henceforth indicates that the mixing matrices have the same structure with respect to the ordinary quark flavors. These relations are justified as follows. The left-handed q-Q mixing matrix $\epsilon_{q_{\mathrm{L}}}$ is calculated as $(\epsilon_{q_{\mathrm{L}}})_{ia} = (\epsilon_{q_{\mathrm{L}}}^{(1)})_{ib}(V_{Q_\chi}^{(2)})_{ba} \sim (m_{q_i}/m_Q)\epsilon_i^f$ with $(V_{Q_\chi}^{(2)})_{ba} \lesssim 1$. The calculation of $\epsilon'_{q_{\mathrm{L}}}$ is made as

$$(\epsilon'_{q_{\mathrm{L}}})_{ia} = (V_{q_{\mathrm{L}}}^{(2)\dagger})_{ii}(\epsilon_{q_{\mathrm{L}}}^{(1)})_{ia} + \sum_{j\neq i}(V_{q_{\mathrm{L}}}^{(2)\dagger})_{ij}(\epsilon_{q_{\mathrm{L}}}^{(1)})_{ja}.$$

The first term is of the order of $(m_{q_i}/m_Q)\epsilon_i^f$ for $(V_{q_{\mathrm{L}}}^{(2)\dagger})_{ii} \sim 1$. The second term is estimated as $(m_{q_i}/m_Q)\epsilon_i^f(\epsilon_j^f)^2$ by using Eqs. (3.141) and (3.142) and the relation

$$\frac{m_{q_i} m_{q_j}}{m_{q_i}^2 + m_{q_j}^2} m_{q_j} < m_{q_i}.$$

It is in fact smaller than the first term. Similar calculations are made for the right-handed q-Q mixing matrices $\epsilon_{q_{\mathrm{R}}}$ and $\epsilon'_{q_{\mathrm{R}}}$. For the left-handed ordinary quark mixing, the relation $(V_{q_{\mathrm{L}}})_{ij} \sim (V_{q_{\mathrm{L}}}^{(2)})_{ij}$ is verified by considering the inequality $(V_{q_{\mathrm{L}}}^{(1)})_{ik}(V_{q_{\mathrm{L}}}^{(2)})_{kj} \lesssim (V_{q_{\mathrm{L}}}^{(1)})_{ij} \lesssim (V_{q_{\mathrm{L}}}^{(2)})_{ij}$ $(k \neq i, j)$ for $m_{q_i} \lesssim m_Q$. The right-handed ordinary quark mixing is estimated with the relation

$$V_{q_{\mathrm{R}}} = V_{q_{\mathrm{R}}}^{(1)} V_{q_{\mathrm{R}}}^{(2)} = V_{q_{\mathrm{R}}}^{(1')} V_{q_{\mathrm{R}}}^{(2'')}, \tag{3.158}$$

where Eqs. (3.148) and (3.154) are considered in the second equality. By using the specific forms (3.148) and (3.153) for $V_{q_{\mathrm{R}}}^{(1')}$ and $V_{q_{\mathrm{R}}}^{(2'')}$, we find the relation $(V_{q_{\mathrm{R}}})_{ij} \sim (V_{q_{\mathrm{R}}}^{(2'')})_{ij}$ under the quark mass hierarchy $m_{q_1} \ll m_{q_2} \ll m_{q_3} \lesssim m_Q$.

Basis (b) with $\Delta_{qQ} = \mathbf{0}$

The diagonalization of $\mathcal{M}_{\mathcal{Q}}$ in the basis (b) with $\Delta_{qQ} = \mathbf{0}$ is practically reduced to what is made in the basis (a) with $\Delta'_{qQ} = \mathbf{0}$. The hermite conjugate of $\mathcal{M}_{\mathcal{Q}}$ with $\Delta_{qQ} = \mathbf{0}$ is given by

$$\mathcal{M}^\dagger_{\mathcal{Q}} = \begin{pmatrix} M^\dagger_q & \Delta'^\dagger_{qQ} \\ \mathbf{0} & M^\dagger_Q \end{pmatrix}. \tag{3.159}$$

This matrix has the same form as $\mathcal{M}_{\mathcal{Q}}$ with $\Delta'_{qQ} = \mathbf{0}$ by replacing the submatrices as

$$\begin{aligned} M_q &\rightarrow M^\dagger_q, \; M_Q \rightarrow M^\dagger_Q, \\ \Delta'_{qQ} = \mathbf{0} &\rightarrow \Delta^\dagger_{qQ} = \mathbf{0}, \; \Delta_{qQ} \rightarrow \Delta'^\dagger_{qQ}. \end{aligned} \tag{3.160}$$

Then, we have the hermite conjugate of Eq. (3.10) as

$$\mathcal{V}^\dagger_{\mathcal{Q}_\mathrm{L}} \mathcal{M}^\dagger_{\mathcal{Q}} \mathcal{V}_{\mathcal{Q}_\mathrm{R}} = \begin{pmatrix} \bar{M}_q & \mathbf{0} \\ \mathbf{0} & \bar{M}_Q \end{pmatrix}. \tag{3.161}$$

Hence, the relations obtained for the basis (a) with $\Delta'_{qQ} = \mathbf{0}$ are also applicable to the present basis (b) with $\Delta_{qQ} = \mathbf{0}$ by taking the substitution (3.160). The q-Q mixing parameters are then replaced as

$$\epsilon^f_i \rightarrow \epsilon^h_i. \tag{3.162}$$

The quark mixing matrices are exchanged as

$$\mathcal{V}_{\mathcal{Q}_\mathrm{L}} \leftrightarrow \mathcal{V}_{\mathcal{Q}_\mathrm{R}}, \tag{3.163}$$

i.e.,

$$\begin{aligned} V_{q_\mathrm{L}} &\leftrightarrow V_{q_\mathrm{R}}, \; V_{Q_\mathrm{L}} \leftrightarrow V_{Q_\mathrm{R}}, \\ \epsilon_{q_\mathrm{L}} &\leftrightarrow \epsilon_{q_\mathrm{R}}, \; \epsilon'_{q_\mathrm{L}} \leftrightarrow \epsilon'_{q_\mathrm{R}}. \end{aligned} \tag{3.164}$$

The ordinary quark masses are given by

$$m_{q_i} = (\lambda_{q_i} v/\sqrt{2}) \left[1 + \xi'_{q_i}(\boldsymbol{\epsilon}^h)(\epsilon^h_i)^2\right] \tag{3.165}$$

with certain factors $\xi'_{q_i}(\boldsymbol{\epsilon}^f) \sim 1$, which are $-1/2$ in the leading order for $N_Q = 1$.

Seesaw model

In the seesaw model, the q-Q mixing matrices at the first step are obtained from Eqs. (3.131) and (3.132) with $M_q = \mathbf{0}$ as

$$\begin{aligned} \epsilon_{q_{\mathrm{L}}}^{(1)} &= \Delta'^{\dagger}_{qQ}(M_Q^\dagger + \delta_{Q_{\mathrm{L}}}^\dagger)^{-1}, && (3.166)\\ \epsilon_{q_{\mathrm{R}}}^{(1)} &= \Delta_{qQ}(M_Q + \delta_{Q_{\mathrm{R}}})^{-1}. && (3.167)\end{aligned}$$

Then, we obtain from Eq. (3.137)

$$M_q^{(1)} = -\Delta_{qQ}(M_Q + \delta_Q)^{-1}\Delta'_{qQ}, \quad (3.168)$$

where

$$\begin{aligned} \delta_Q &= (C_Q \epsilon_{q_{\mathrm{R}}}^{(1)\dagger}\Delta_{qQ} - \delta_{Q_{\mathrm{L}}})[\mathbf{1} + \mathcal{E}_Q \epsilon_{q_{\mathrm{R}}}^{(1)\dagger}\Delta_{qQ}]^{-1} \\ &\sim \Delta'_{qQ}\epsilon_{q_{\mathrm{L}}}^{(1)} + \epsilon_{q_{\mathrm{R}}}^{(1)\dagger}\Delta_{qQ} && (3.169)\end{aligned}$$

with

$$\begin{aligned} C_Q &= M_Q(M_Q + \delta_{Q_{\mathrm{R}}})^{-1}(V_{Q_{\mathrm{R}}}^{(1)\dagger -1} - A_{q_{\mathrm{R}}}), && (3.170)\\ \mathcal{E}_Q &= (M_Q + \delta_{Q_{\mathrm{R}}})^{-1}(V_{Q_{\mathrm{R}}}^{(1)\dagger -1} - A_{q_{\mathrm{R}}}). && (3.171)\end{aligned}$$

Now suppose that all the singlet quarks have comparable masses, $m_{Q_1},\ m_{Q_2},\ m_{Q_3} \sim m_Q$. Then, we obtain from Eqs. (3.166) and (3.167)

$$\begin{aligned} (\epsilon_{q_{\mathrm{L}}}^{(1)})_{ia} &\sim \epsilon_i^h, && (3.172)\\ (\epsilon_{q_{\mathrm{R}}}^{(1)})_{ia} &\sim \epsilon_i^f, && (3.173)\\ (V_{q_{\mathrm{L}}}^{(1)})_{ij} &\sim \delta_{ij} + \epsilon_i^h\epsilon_j^h, && (3.174)\\ (V_{q_{\mathrm{R}}}^{(1)})_{ij} &\sim \delta_{ij} + \epsilon_i^f\epsilon_j^f. && (3.175)\end{aligned}$$

We can also see from Eq. (3.168) that the effective quark mass matrix has a specific flavor structure,

$$(M_q^{(1)})_{ij} \sim \epsilon_i^f\epsilon_j^h m_Q. \quad (3.176)$$

The unitary matrices to diagonalize $M_q^{(1)}$ are given as

$$\begin{aligned} (V_{q_{\mathrm{L}}}^{(2)})_{ij} &\sim \delta_{ij} + \frac{\epsilon_i^h\epsilon_j^h}{(\epsilon_i^h)^2 + (\epsilon_j^h)^2}, && (3.177)\\ (V_{q_{\mathrm{R}}}^{(2)})_{ij} &\sim \delta_{ij} + \frac{\epsilon_i^f\epsilon_j^f}{(\epsilon_i^f)^2 + (\epsilon_j^f)^2}. && (3.178)\end{aligned}$$

Then, the net effects on the quark mixings are calculated as

$$\epsilon_{q_\chi},\ \epsilon'_{q_\chi} \sim \epsilon^{(1)}_{q_\chi}\ (\chi = \mathrm{L}, \mathrm{R}), \tag{3.179}$$

$$V_{q_\chi} \sim V^{(2)}_{q_\chi}\ (\chi = \mathrm{L}, \mathrm{R}). \tag{3.180}$$

Here, the inequality $(V^{(1)}_{q_\chi})_{ik}(V^{(2)}_{q_\chi})_{kj} \lesssim (V^{(1)}_{q_\chi})_{ij} \lesssim (V^{(2)}_{q_\chi})_{ij}$ $(k \neq i, j)$ for $\epsilon^f_i, \epsilon^h_i \lesssim 1$ is considered. The ordinary quark masses are obtained as

$$m_{q_i} \sim \epsilon^f_i \epsilon^h_i m_Q. \tag{3.181}$$

We next consider the case with inverted hierarchy for the singlet quark masses, $m_{Q_1} \gg m_{Q_2} \gg m_{Q_3}$ [39]. The q-Q mixing terms are assumed to have no significant flavor dependence, i.e., $(\Delta_{qQ})_{ia} \sim \bar{f} v_S$ and $(\Delta'_{qQ})_{ai} \sim \bar{h} v$. We may start with

$$M_Q = \mathrm{diag.}(m^0_{Q_1}, m^0_{Q_2}, m^0_{Q_3}) \tag{3.182}$$

by using suitable transformations of singlet quarks. The singlet quark masses are provided dominantly by this M_Q term for $m^0_{Q_a} \gg \bar{f} v_S,\ \bar{h} v$:

$$m_{Q_a} \simeq m^0_{Q_a}. \tag{3.183}$$

(This relation is replaced by that of $m_T \sim \bar{f} v_S$ for the singlet quark $T \equiv U_3$ with $m^0_T \ll \bar{f} v_S$ [39].) It is also suitable to deform the q-Q mixing terms to triangular forms by the ordinary quark transformations, without modifying $M_q = \mathbf{0}$ and M_Q:

$$\Delta_{qQ} = \bar{f} v_S \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}, \tag{3.184}$$

$$\Delta'_{qQ} = \bar{h} v \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.185}$$

where "1" denotes the factors of $O(1)$.

The effective quark mass matrix $M^{(1)}_q$ given in Eq. (3.168) is evaluated as follows. We first note the identity with the diagonal M_Q,

$$(M_Q + \delta_Q)^{-1}_{ab} = m^{0-1}_{Q_a} \delta_{ab} - m^{0-1}_{Q_a} \left[\delta_Q (\mathbf{1} + M_Q^{-1} \delta_Q)^{-1} \right]_{ab} m^{0-1}_{Q_b}, \tag{3.186}$$

where, as seen from Eq. (3.169) with $\epsilon^{(1)}_{q_\chi} \lesssim 1$ and Eqs. (3.184) and (3.185),

$$\left| \left[\delta_Q (\mathbf{1} + M_Q^{-1} \delta_Q)^{-1} \right]_{ab} \right| \lesssim \bar{f} v_S + \bar{h} v. \tag{3.187}$$

Then, by applying Eqs. (3.184), (3.185) and (3.186) for Eq. (3.168), the effective quark mass matrix $M_q^{(1)}$ is evaluated in terms of $m_{Q_a}^{-1} \simeq m_{Q_a}^{0-1}$ as

$$M_q^{(1)} \sim (\bar{h}v)(\bar{f}v_S) \begin{pmatrix} m_{Q_1}^{-1} & m_{Q_1}^{-1} & m_{Q_1}^{-1} \\ m_{Q_1}^{-1} & m_{Q_2}^{-1} & m_{Q_2}^{-1} \\ m_{Q_1}^{-1} & m_{Q_2}^{-1} & m_{Q_3}^{-1} \end{pmatrix}. \tag{3.188}$$

The second term in the right-hand side of Eq. (3.186) actually provides sub-leading contributions as $(\bar{h}v)(\bar{f}v_S)^2 m_{Q_i}^{-1} m_{Q_j}^{-1}$. These corrections, however, do not alter the structure (3.188) of $M_q^{(1)}$ for $\bar{f}v_S/m_{Q_i} \lesssim 1$. This specific form of $M_q^{(1)}$ provides the ordinary quark masses,

$$m_{q_i} \sim (\bar{f}v_S/m_{Q_i})\bar{h}v. \tag{3.189}$$

The first step mixings are obtained from Eqs. (3.166) and (3.167) by using Eqs. (3.184) and (3.185) for Δ_{qQ} and Δ'_{qQ} and the relation similar to Eq. (3.186):

$$(\epsilon_{q_\mathrm{L}}^{(1)})_{ia} \sim \bar{h}v/m_{Q_i}, \tag{3.190}$$

$$(\epsilon_{q_\mathrm{R}}^{(1)})_{ia} \sim \bar{f}v_S/m_{Q_i}, \tag{3.191}$$

$$(V_{q_\mathrm{L}}^{(1)})_{ij} \sim \delta_{ij} + \frac{(\bar{h}v)^2}{m_{Q_i} m_{Q_j}}, \tag{3.192}$$

$$(V_{q_\mathrm{R}}^{(1)})_{ij} \sim \delta_{ij} + \frac{(\bar{f}v_S)^2}{m_{Q_i} m_{Q_j}}. \tag{3.193}$$

The unitary transformations to diagonalize $M_q^{(1)}$ of Eq. (3.188) are given by

$$(V_{q_\mathrm{L}}^{(2)})_{ij},\ (V_{q_\mathrm{R}}^{(2)})_{ij} \sim \delta_{ij} + \frac{m_{Q_i}^{-1} m_{Q_j}^{-1}}{m_{Q_i}^{-2} + m_{Q_j}^{-2}}. \tag{3.194}$$

It is here noticed that these relations (3.190) – (3.194) for the quark mixings are apparently reproduced from Eqs. (3.172) – (3.178) for the case of $m_{Q_1}, m_{Q_2}, m_{Q_3} \sim m_Q$ by taking the substitution

$$\epsilon_i^h \to \bar{h}v/m_{Q_i},\ \epsilon_i^f \to \bar{f}v_S/m_{Q_i}. \tag{3.195}$$

Hence, even in this case of $m_{Q_1} \ll m_{Q_2} \ll m_{Q_3}$, the net effects on the quark mixings are given as Eqs. (3.179) and (3.180). It is interesting that these quark mixings can be expressed in term of the ordinary quark masses by using the mass formula (3.189).

Some remarks should finally be presented in order. As for the t and T quarks with $(M_U)_{33} = m_T^0 \ll \bar{f}v_S$, the above arguments are still valid to obtain $m_t \sim \bar{h}v$ and $m_T \sim \bar{f}v_S$. This can be understood by modifying the mass matrix in Eq. (3.186) as $M_U + \delta_U =$

$\tilde{M}_U + \tilde{\delta}_U$ with $(\tilde{M}_U)_{33} = (M_U)_{33} + (\delta_U)_{33} \sim \bar{f} v_S$. In the general quark basis for the seesaw model, the Δ_{qQ} and Δ'_{qQ} terms do not have the triangular forms (3.184) and (3.185). Then, the following substitution should be made in the above relations for the quark mixings,

$$\epsilon_{q_\chi} \to \tilde{V}^\dagger_{q_\chi} \epsilon_{q_\chi}, \; V_{q_\chi} \to \tilde{V}^\dagger_{q_\chi} V_{q_\chi}. \tag{3.196}$$

Here, $\tilde{V}^\dagger_{q_{\mathrm{R}}} \Delta_{qQ}$ and $\Delta'_{qQ} \tilde{V}_{q_{\mathrm{L}}}$ become the triangular forms (3.184) and (3.185) by suitably choosing the unitary transformations $\tilde{V}_{q_\chi}$.

Chapter 4

Universality of strength for Yukawa couplings with extra down-type quark singlets

In this chapter, we investigate the quark masses and mixings by including vector-like down-type quark singlets in universality of strength for Yukawa couplings (USY). In contrast with the standard model with USY, the sufficient CP violation is obtained for the Cabibbo-Kobayashi-Maskawa matrix through the mixing between the ordinary quarks and quark singlets. The top-bottom mass hierarchy $m_t \gg m_b$ also appears naturally in the USY scheme with the down-type quark singlets.

4.1 Introduction

The explanation of the masses and mixings of quarks and leptons is one of the fundamental issues in particle physics. Many notable ideas to address this problem have been investigated, including the universality of strength for Yukawa couplings (USY) [44, 45, 46, 47, 48, 49, 50]. In the standard model with USY, the nearly democratic quark mass matrices [51] (see also references therein) are provided, and the quark masses and the magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are really reproduced with the suitable USY phases. However, the USY scheme seems to confront some difficulties within the context of the standard model. Some reasonable explanation should be presented for the top-bottom mass hierarchy $m_t \gg m_b$; it is simply attributed to the hierarchy of the Yukawa couplings between the up and down sectors with one Higgs doublet, or a large ratio of the vacuum expectation values (VEV's) of two Higgs doublets. More seriously, it is quite difficult to obtain the sufficient CP violation for the CKM matrix in the standard model with USY [44, 45, 46, 47, 52], which is essentially due to the fact that the USY phases are small to provide the quark masses except for the third generation.

In this chapter, we present a new look at the USY scheme by including exotic ingredients. Specifically, we investigate an extension of the standard model with extra down-type quark singlets [6, 10, 14, 16, 26] (see also references therein). The standard model contains three generations of the ordinary quarks, left-handed doublets $q_{i\mathrm{L}} = (u_{i\mathrm{L}}, d_{i\mathrm{L}})^{\mathrm{T}}$ and right-handed singlets $u_{i\mathrm{R}}$, $d_{i\mathrm{R}}$ ($i = 1, 2, 3$), and a Higgs doublet H. In addition, N_D vector-like down-type quark singlets $D_{a\mathrm{L}}$ and $D_{a\mathrm{R}}$ ($a = 4, \ldots, 3 + N_D$) and a Higgs singlet S are included [10, 26], which may be accommodated in E_6-type models [2, 3, 4]. We will show that the actual quark masses and CKM matrix are indeed obtained in the USY scheme with extra down-type quark singlets. In particular, through the d-D mixing the sufficient CP violation for the CKM matrix is provided from some large USY phases of the Yukawa couplings with the Higgs singlet S. (This mixing mechanism to transmit the CP violation is considered in Refs. [6, 10, 16].) The top-bottom hierarchy $m_t \gg m_b$ also appears naturally in the USY scheme (or more generally flavor democracy) due to the existence of extra down-type quark singlets but no such up-type quark singlets as in the **27** of E_6.

4.2 Quark masses and mixings with extra down-type quarks in USY

The Yukawa couplings of quarks and Higgs fields with USY are given by

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Y}} &= -\bar{q}_{i\mathrm{L}} \Lambda^u_{ij} u_{j\mathrm{R}} H - \bar{q}_{i\mathrm{L}} \Lambda^d_{iJ} \mathcal{D}_{J\mathrm{R}} (i\tau_2 H^*) \\
&\quad - \bar{D}_{a\mathrm{L}} \Lambda^D_{aJ} \mathcal{D}_{J\mathrm{R}} S + \mathrm{H.c.}, \\
\Lambda^u_{ij} &= \frac{\lambda_u}{3} \mathrm{e}^{i\phi^u_{ij}}, \Lambda^d_{iJ} = \frac{\lambda_d}{3} \mathrm{e}^{i\phi^d_{iJ}}, \Lambda^D_{aJ} = \frac{\lambda_D}{3} \mathrm{e}^{i\phi^D_{aJ}},
\end{aligned}
\tag{4.1}
$$

where $J = j, b$ with $\mathcal{D}_j \equiv d_j$ and $\mathcal{D}_b \equiv D_b$. The respective types of Yukawa couplings are specified with the strengths λ_u, λ_d, λ_D and USY phases ϕ^u_{ij}, ϕ^d_{iJ}, ϕ^D_{aJ}. The couplings $\bar{D}_{a\mathrm{L}} \mathcal{D}_{J\mathrm{R}} S^*$ are excluded here for definiteness if S is a complex field. This is really the case for the supersymmetric model with a pair of Higgs doublets. The quark mass matrices are given from Eq. (4.1) as

$$
M_u = (\lambda v/3) \left(\mathrm{e}^{i\phi^u_{ij}} \right), \mathcal{M}_{\mathcal{D}} = (\lambda v/3) \begin{pmatrix} \mathrm{e}^{i\phi^d_{iJ}} \\ \kappa \mathrm{e}^{i\phi^D_{aJ}} \end{pmatrix}, \tag{4.2}
$$

where $\langle H^0 \rangle = v$, $\langle S \rangle = v_S$ (the possible phase is absorbed by ϕ^D_{aJ}), and $\kappa = v_S/v$. We investigate the case of $\lambda_u = \lambda_d = \lambda_D = \lambda$ for definiteness, while the result is readily extended for different λ_u, λ_d, λ_D.

We first consider the up-type quark mass matrix

$$M_u = M_u(0) + \Delta M_u = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_u, \tag{4.3}$$

where the perturbation part is given as $\Delta M_u \simeq i\Phi_u$ with the small USY phase matrix $(\Phi_u)_{ij} = \phi^u_{ij}$. Henceforth the quark mass terms are presented to be dimensionless measured in unit of $\lambda v/3$ ($\simeq m_t/3$). The up-type quark mass matrix is relevantly expressed in the hierarchical basis by making a suitable transformation [44, 45, 46, 47, 48, 49, 50]:

$$\widetilde{M}_u = U_q^\dagger M_u U_q I_u \simeq \mathrm{diag}(0,0,3) + i\widetilde{\Phi}_u I_u, \tag{4.4}$$

where $\widetilde{\Phi}_u = U_q^\dagger \Phi_u U_q$, $U_q = U(1)_{[12]} U(\sqrt{2})_{[23]}$, $I_u = \mathrm{diag}(-i,-i,1)$, and "diag" represents a diagonal matrix. The unitary transformation $U(\alpha)_{[IJ]}$ between the I-th and J-th quarks is specified with a 2×2 matrix

$$U(\alpha) = \frac{1}{\sqrt{1+|\alpha|^2}} \begin{pmatrix} 1 & \alpha \\ -\alpha^* & 1 \end{pmatrix},$$

supplemented with the right dimension, 3×3 for the up sector and $(3+N_D)\times(3+N_D)$ for the down sector. We note here that by suitably choosing the phases of $q_{i\mathrm{L}}$'s and $u_{j\mathrm{R}}$'s, the USY phases are taken in general as $\phi^u_{i3} = -\phi^u_{i1} - \phi^u_{i2}$ and $\phi^u_{3j} = -\phi^u_{1j} - \phi^u_{2j}$, giving $(\widetilde{\Phi}_u)_{i3} = (\widetilde{\Phi}_u)_{3j} = 0$. In this USY phase convention, the pre-factor i for $\tilde{\Phi}_u$ is practically removed with I_u, and the up-type quark mass matrix in Eq. (4.4) is given as

$$\widetilde{M}_u = V_{u\mathrm{L}} \mathrm{diag}(m_u, m_c, m_t) V^\dagger_{u\mathrm{R}} \simeq \begin{pmatrix} \tilde{\phi}_{u1} & \tilde{\phi}_{u2} & 0 \\ \tilde{\phi}_{c1} & \tilde{\phi}_{c2} & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{4.5}$$

with $\tilde{\phi}_{u_i j} = (\widetilde{\Phi}_u)_{ij}$ ($u_1 = u, u_2 = c, u_3 = t$).

The quark mass hierarchy $m_u \ll m_c \ll m_t$ for the nearly democratic M_u in Eq. (4.3) is understood in terms of the sequential breakings of the permutation symmetry $S^3_{q\mathrm{L}}$ among the left-handed quark doublets [53]:

$$S^3_{q\mathrm{L}} \to S^2_{q\mathrm{L}} \to \mathrm{non}. \tag{4.6}$$

The democratic and $S^3_{q\mathrm{L}}$ invariant $M_u(0)$ provides the top mass. Then, for the USY phases in Eq. (4.5) the $S^2_{q\mathrm{L}}$ invariant terms $\tilde{\phi}_{cj}$ and the small $S^2_{q\mathrm{L}}$ breaking ones $\tilde{\phi}_{uj}$ provide the charm and up masses, respectively, as

$$|\tilde{\phi}_{cj}| \sim m_c \gg |\tilde{\phi}_{uj}| \sim m_u \tag{4.7}$$

with $(V_{u\mathrm{L}})_{12} \sim m_u/m_c \ll 1$ ($V_{u\mathrm{L}} \simeq \mathbf{1}$).

We next investigate the down sector including two singlet D's, while the essential features are valid for $N_D \geq 2$. The USY scheme with only one D is, however, unsatisfactory, still providing the too small CP violation for the CKM matrix. This is because the USY phases ϕ^D_{4J} in Λ^D with the Higgs singlet S are all eliminated away by rephasing $\mathcal{D}_{J\mathrm{R}}$'s. Then, the remaining USY phases should be small to provide the ordinary quark masses just as in the standard model with USY.

The down-type quark mass matrix is given as

$$\mathcal{M}_{\mathcal{D}} = \mathcal{M}_{\mathcal{D}}(0) + \Delta\mathcal{M}_{\mathcal{D}}. \tag{4.8}$$

The main part has a quasi-democratic form

$$\mathcal{M}_{\mathcal{D}}(0) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ \kappa & \kappa & \kappa & \kappa & \kappa \\ \kappa & \kappa & \kappa & \kappa & \kappa \end{pmatrix} \tag{4.9}$$

with $\kappa = v_S/v$. The remaining part $\Delta\mathcal{M}_{\mathcal{D}}$ is provided with the USY phase matrix $\Phi_{\mathcal{D}}$, $(\Phi_{\mathcal{D}})_{iJ} = \phi^d_{iJ}$ and $(\Phi_{\mathcal{D}})_{aJ} = \phi^D_{aJ}$. In accordance with Eq. (4.4) for the up sector, the mass matrix of down sector is transformed as

$$\begin{aligned} \widetilde{\mathcal{M}}_{\mathcal{D}} &= U(1)^\dagger_{[45]} U^\dagger_q \mathcal{M}_{\mathcal{D}} U_q U(\sqrt{3})_{[34]} U(2)_{[45]} I_{\mathcal{D}} \\ &= \begin{pmatrix} \widetilde{M}_d & \widetilde{\Delta}'_{dD} \\ \widetilde{\Delta}_{dD} & \widetilde{M}_D \end{pmatrix}, \end{aligned} \tag{4.10}$$

where $I_{\mathcal{D}} = \mathrm{diag}(-i, -i, -i, -\mathrm{e}^{-i\theta}, 1)$ with θ to be fixed below in Eq. (4.14). The USY phase matrix $\Phi_{\mathcal{D}}$ is transformed in the same way to $\widetilde{\Phi}_{\mathcal{D}} I_{\mathcal{D}}$. This transformation respects the $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge symmetry without d_L-D_L mixing. The main part is given in this basis as

$$\widetilde{\mathcal{M}}_{\mathcal{D}}(0) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{15} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{10}\kappa \end{pmatrix}, \tag{4.11}$$

providing four $(3+N_D-1)$ zero eigenvalues. Hence, in contrast with the flavor democracy in the standard model, the bottom quark no longer acquires so a heavy mass as the top quark. This reasonably explains the top-bottom hierarchy $m_t \gg m_b$ in the USY scheme. It is also noticed that one (N_D-1) D should obtain a mass from the USY phases as well as the ordinary d's.

The USY phases in Λ^d with the Higgs doublet H are supposed to be small to provide the ordinary quark masses and mixings. On the other hand, those in Λ^D with the Higgs singlet S may be large to provide the significant CP violation for the CKM matrix through the d-D mixing. It is convenient here to make $\phi^D_{5J}=0$ by rephasing $\mathcal{D}_{J\mathrm{R}}$'s. We may also take for simplicity $\phi^D_{4j} \approx \phi^D_{44} \approx 0$ under the approximate $S^4_{\mathcal{D}\mathrm{R}}$ among $\mathcal{D}_{1\mathrm{R}} - \mathcal{D}_{4\mathrm{R}}$ together with the rephasing of $\mathcal{D}_{4\mathrm{L}}$ (though not essential for the desired CP violation). That is, in this convention

$$|\phi^d_{iJ}|, |\phi^D_{4j}|, |\phi^D_{44}| \ll 1, |\phi^D_{45}| \sim 1, \phi^D_{5J}=0. \tag{4.12}$$

The submatrix $\widetilde{M}_D$ in $\widetilde{\mathcal{M}}_{\mathcal{D}}$ is given as

$$\widetilde{M}_D \simeq \frac{\kappa}{\sqrt{10}} \begin{pmatrix} 2|\Delta| & \Delta \\ 2|\Delta| & 10+\Delta \end{pmatrix}, \tag{4.13}$$

where

$$\Delta \equiv |\Delta| \mathrm{e}^{i\theta} \equiv \exp[i\phi^D_{45}] - 1 \tag{4.14}$$

with $\theta = [(|\phi^D_{45}| + \pi)/2]\mathrm{Sign}[\phi^D_{45}]$. Then, the masses of the heavy quarks, almost the singlets, are given as

$$m_{D_1} \simeq (2/\sqrt{10})|\Delta|\kappa, m_{D_2} \simeq \sqrt{10}\kappa. \tag{4.15}$$

The submatrix $\widetilde{M}_d$ for the ordinary quarks is given as

$$\begin{aligned} \widetilde{M}_d &= V^{(0)}_{d\mathrm{L}} \mathrm{diag}(m^{(0)}_d, m^{(0)}_s, m^{(0)}_b) V^{(0)\dagger}_{d\mathrm{R}} \\ &\simeq \widetilde{\Phi}^{(3)}_{\mathcal{D}} = \begin{pmatrix} \tilde{\phi}_{d1} & \tilde{\phi}_{d2} & \tilde{\phi}_{d3} \\ \tilde{\phi}_{s1} & \tilde{\phi}_{s2} & \tilde{\phi}_{s3} \\ \tilde{\phi}_{b1} & \tilde{\phi}_{b2} & \tilde{\phi}_{b3} \end{pmatrix} \end{aligned} \tag{4.16}$$

with $m^{(0)}_{d_i} \sim m_{d_i}$ $(d_1=d, d_2=s, d_3=b)$, where $\widetilde{\Phi}^{(3)}_{\mathcal{D}}$ is the 3×3 submatrix of $\widetilde{\Phi}_{\mathcal{D}}$. (The pre-factor i is removed for $\widetilde{\Phi}^{(3)}_{\mathcal{D}}$ with $I_{\mathcal{D}}$.) In accordance with the up sector, the hierarchical quark masses and mixings may be reproduced in terms of $S^3_{q\mathrm{L}}$ and $S^2_{q\mathrm{L}}$ in Eq. (4.6) as

$$|\tilde{\phi}_{bj}| \sim m_b \gg |\tilde{\phi}_{sj}| \sim m_s \gg |\tilde{\phi}_{dj}| \sim m_d. \tag{4.17}$$

The left-handed mixing $V_{d\mathrm{L}}^{(0)}$ in Eq. (4.16) is taken as the pre-CKM matrix (Particle Data Group convention) with the vanishing complex phase $\delta_{13}^{(0)} \simeq 0$ and the mixing angles $\theta_{ij}^{(0)} \sim m_{d_i}/m_{d_j}$ $(i < j)$ from Eq. (4.17). Then, by including the d-D mixing effects as seen below, the CKM matrix with sufficient CP violation can be reproduced with reasonable values of $\theta_{ij}^{(0)}$, which are adjustable in terms of the USY phases. The right-handed mixing $V_{d\mathrm{R}}^{(0)}$, on the other hand, may be absorbed practically into $d_{j\mathrm{R}}$'s without physical effects.

The d-D mixing terms in Eq. (4.10) are given as

$$\widetilde{\Delta}_{dD} \simeq \kappa \begin{pmatrix} \tilde{\phi}_{D_1 d} & \tilde{\phi}_{D_1 s} & \tilde{\phi}_{D_1 b} \\ \tilde{\phi}_{D_2 d} & \tilde{\phi}_{D_2 s} & \tilde{\phi}_{D_2 b} \end{pmatrix}, \tag{4.18}$$

$$\widetilde{\Delta}'_{dD} \simeq \begin{pmatrix} -i\mathrm{e}^{-i\theta}\tilde{\phi}_{dD_1} & i\tilde{\phi}_{dD_2} \\ -i\mathrm{e}^{-i\theta}\tilde{\phi}_{sD_1} & i\tilde{\phi}_{sD_2} \\ -i\mathrm{e}^{-i\theta}\tilde{\phi}_{bD_1} & i\tilde{\phi}_{bD_2} + \sqrt{15} \end{pmatrix}, \tag{4.19}$$

where $\tilde{\phi}_{D_k d_j} = (\widetilde{\Phi}_{\mathcal{D}})_{3+k,j}$ and $\tilde{\phi}_{d_i D_k} = (\widetilde{\Phi}_{\mathcal{D}})_{i,3+k}$. These d-D mixing terms provide certain corrections to $\widetilde{M}_d$, which may be evaluated perturbatively as

$$(\delta\widetilde{M}_d)_{ij} \simeq -\sum_{D_k} (\widetilde{\Delta}'_{dD})_{i4} (\widetilde{\Delta}_{dD})_{4j} / m_{D_k}. \tag{4.20}$$

Then, mainly through D_1, significant imaginary parts are provided to V_{ub} and V_{td} for the desired CP violation as

$$\begin{aligned} \mathrm{Im}[V_{ub}] &\simeq \mathrm{Im}[V_{td}] \simeq \mathrm{Im}[(\delta\widetilde{M}_d)_{13}/(\widetilde{M}_d)_{33}] \\ &\simeq \sqrt{\frac{5}{2}} \frac{\tilde{\phi}_{dD_1}\tilde{\phi}_{D_1 b}}{\tilde{\phi}_{b3}} \frac{\cos\theta}{|\Delta|} \sim -0.003. \end{aligned} \tag{4.21}$$

In total, the left-handed mixing $V_{d\mathrm{L}}$ for the ordinary $d_{i\mathrm{L}}$'s is determined as the 3×3 submatrix of the unitary matrix to diagonalize the entire $\widetilde{\mathcal{M}}_{\mathcal{D}}$ in Eq. (4.10) [6, 10, 14, 16, 26] (see also references therein). Then, the weak charged current mixing matrix V (CKM matrix) for the ordinary quarks is given ($V_{u\mathrm{L}} \simeq \mathbf{1}$) by

$$V = V_{u\mathrm{L}}^{\dagger} V_{d\mathrm{L}}. \tag{4.22}$$

Here, the case of diagonal $\widetilde{M}_d$ in Eq. (4.16) ($V_{d\mathrm{L}}^{(0)} = V_{d\mathrm{R}}^{(0)} = \mathbf{1}$) may be specifically interesting, where the CKM mixing emerges entirely from the d-D mixing in the hierarchical basis. In this case, V_{us}, in particular, is estimated as

$$|V_{us}| \simeq \frac{|(\delta\widetilde{M}_d)_{12}|}{|m_s^{(0)} + (\delta\widetilde{M}_d)_{22}|} \lesssim \frac{|V_{ub}|/|V_{cb}|}{|\cos\theta|}, \tag{4.23}$$

where the relations,

$$\begin{aligned}|m_s^{(0)} + (\delta \widetilde{M}_d)_{22}| &\geq |\mathrm{Im}[(\delta \widetilde{M}_d)_{22}]|, \\ |\tilde{\phi}_{dD_1}|/|\tilde{\phi}_{sD_1}| &\simeq |(\delta \widetilde{M}_d)_{1j}|/|(\delta \widetilde{M}_d)_{2j}| \simeq |V_{ub}|/|V_{cb}|,\end{aligned}$$

are considered.

The d-D mixing also induces small corrections to the weak neutral currents, which are related to the unitarity violation of $V_{d\mathrm{L}}$ [6, 10, 14, 16, 26] (see also references therein). We estimate, in particular,

$$\begin{aligned}|(V_{d\mathrm{L}}^\dagger V_{d\mathrm{L}})_{33} - 1| &\simeq |(V_{d\mathrm{L}} V_{d\mathrm{L}}^\dagger)_{33} - 1| \\ &\simeq |(\widetilde{\Delta}'_{dD})_{35}(\Delta/5)|^2/m_{D_1}^2 + |(\widetilde{\Delta}'_{dD})_{35}|^2/m_{D_2}^2 \\ &\simeq 3/\kappa^2,\end{aligned}$$

where the correction to $(\widetilde{\Delta}'_{dD})_{34}$ through the $D_{1\mathrm{R}}$-$D_{2\mathrm{R}}$ mixing $\simeq |\Delta/5|$ is included. Then, in order to suppress the correction to R_b for $Z \to b\bar{b}$ to be less than 0.1 %,

$$\kappa = v_S/v \gtrsim 50 \tag{4.24}$$

is required, implying $m_{D_1} \gtrsim 1\mathrm{TeV}$ with $|\Delta| \gtrsim 0.5$. This hierarchy of the VEV's may be realized naturally in some supersymmetric model with an extra gauge symmetry ($\subset \mathrm{E}_6$) spontaneously broken by $\langle S \rangle = v_S$. The quark singlet with $m_{D_1} \sim 1\mathrm{TeV}$ may provide a sizable contribution to the neutron electric dipole moment, while the effect on ϵ'/ϵ will be small enough [6, 10, 16].

4.3 Numerical result for the CKM matrix

A numerical result is obtained for the CKM matrix with the CP violation angles as

$$|V| = \begin{pmatrix} 0.9743 & 0.2253 & 0.0035 \\ 0.2252 & 0.9735 & 0.0410 \\ 0.0088 & 0.0403 & 0.9986 \end{pmatrix},$$

$$\alpha = 87.1^\circ, \beta = 21.5^\circ, \gamma = 71.5^\circ,$$

and the rephasing invariant CP violation measure $J = 2.99 \times 10^{-5}$. The USY phases are taken suitably with $\kappa = 50$; $\phi^u_{ij} = 3[U_q \mathrm{diag}(m_u/m_t, m_c/m_t, 0) U_q^\dagger]_{ij}$ for $\widetilde{M}_u$ ($\widetilde{\Phi}_u$) in Eq. (4.5) with $V_{u\mathrm{L}} = V_{u\mathrm{R}} = \mathbf{1}$; ϕ^d_{i2}, ϕ^d_{i3}, ϕ^d_{i4} with $\phi^d_{i1} = 0$ for $\widetilde{M}_d$ ($\widetilde{\Phi}^{(3)}_{\mathcal{D}}$) in Eq.

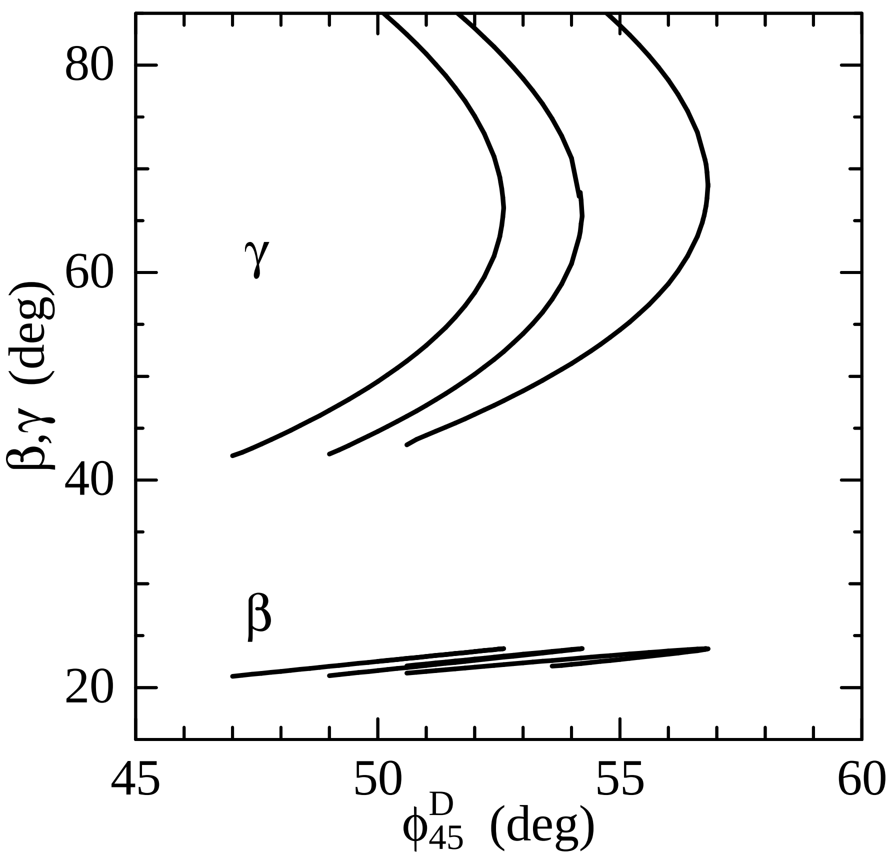

Figure 4.1: The CP violation angles β and γ of the CKM matrix versus the USY phase ϕ_{45}^{D} are shown, where $\tilde{\phi}_{dD}$ is taken typically as -0.003 (right), -0.01 (left), -0.02 (middle).

(4.16) with $V_{d\mathrm{L}}^{(0)} = V_{d\mathrm{R}}^{(0)} = \mathbf{1}$ and $(m_d^{(0)}/m_d, m_s^{(0)}/m_s, m_b^{(0)}/m_b) = (1.026, 2.482, 1.008)$; $\phi^d_{i5} = (U_q)_{ij}\tilde{\phi}_{d_jD}$ with $(\tilde{\phi}_{dD}, \tilde{\phi}_{sD}, \tilde{\phi}_{bD}) = (-0.01, -0.118, 0)$ for $\widetilde{\Delta}'_{dD}$; $(\phi^D_{41}, \phi^D_{42}, \phi^D_{43}, \phi^D_{44}, \phi^D_{45}) = (0, 0, 0.0386, -0.0116, 0.840)$ with $\phi^D_{5J} = 0$ for $\widetilde{\Delta}_{dD}$ and $\widetilde{M}_D$. The quark masses are obtained as

$$
\begin{aligned}
&m_u = 3\mathrm{MeV}, m_c = 1.25\mathrm{GeV}, m_t = 173\mathrm{GeV},\\
&m_d = 5\mathrm{MeV}, m_s = 100\mathrm{MeV}, m_b = 4.25\mathrm{GeV},\\
&m_{D_1} = 1.50\mathrm{TeV}, m_{D_2} = 9.00\mathrm{TeV}
\end{aligned}
$$

with $m_{D_1}/m_{D_2} \simeq |\Delta|/5$ ($|\Delta| = 0.82$). The result of a USY phase space scan is also shown in Fig. 4.1 for the CP violation angles β (lower) and γ (upper) versus the USY phase ϕ^D_{45}. The USY phase values are taken as in the above example. In particular, $\tilde{\phi}_{dD}$ is taken typically as -0.003 (right), -0.01 (left), -0.02 (middle) with $\tilde{\phi}_{bD} = 0$, by considering $|\tilde{\phi}_{D_1b}| \lesssim 0.1$ in Eq. (4.21) with $\tilde{\phi}_{b3} \simeq m_b/(m_t/3)$, and $|\tilde{\phi}_{sD_1}| \simeq (|V_{cb}|/|V_{ub}|)|\tilde{\phi}_{dD_1}| \lesssim 0.1$. Then, $\tilde{\phi}_{sD}$, ϕ^D_{43}, ϕ^D_{44} ($\phi^D_{41} = \phi^D_{42} = 0$) and $m_s^{(0)}/m_s$ are adjusted for $|V_{us}| = 0.225$, $|V_{cb}| = 0.041$ and $|V_{ub}| = 0.0035$. (The small $\tilde{\phi}_{bD}$ may be eliminated by rephasing $D_{5\mathrm{R}}$, which is almost compensated with a slight shift of ϕ^D_{45} by $\tilde{\phi}_{bD}/\sqrt{3} \sim$ some degree.) We have found suitable USY phase values to reproduce the quark masses and CKM matrix with $\beta \simeq 21^\circ - 24^\circ$ and $\gamma \simeq 40^\circ - 90^\circ$. This range of β is really reproduced by the Particle Data Group convention with $\gamma \simeq \delta_{13}$ and $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ for θ_{12}, θ_{23}, θ_{13}. (Solutions are not found for $\gamma \lesssim 40^\circ$ or $\gtrsim 90^\circ$ with our computation algorithm.) As seen in Fig. 4.1, ϕ^D_{45} takes the maximal value $\bar{\phi}^D_{45}$ for given $\tilde{\phi}_{dD}$, providing $\gamma = \bar{\gamma}$. (We have evaluated $\bar{\gamma} \simeq 66^\circ - 71^\circ$ and $\bar{\phi}^D_{45} \simeq 53^\circ - 62^\circ$ for $-0.03 \leq \tilde{\phi}_{dD} \leq -0.002$.) This corresponds to the condition $m_s^{(0)} + \mathrm{Re}[(\delta\widetilde{M}_d)_{22}] = 0$ in Eq. (4.23), specifying $\bar{\gamma} \simeq \pi - \bar{\theta} = (\pi - \bar{\phi}^D_{45})/2$, as verified by calculating V_{d_L} roughly with Eq. (4.20). The mass of the lighter quark singlet is estimated as $m_{D_1} \approx 1.5\mathrm{TeV}(\kappa/50)$ for $\phi^D_{45} \approx 50^\circ$. These results are valid for the experimentally determined range of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$.

4.4 Discussion and conclusion

The USY structure may be realized just above the electroweak scale as in some large extra dimension models [54, 55]. On the other hand, if it is given at a very high unification scale, the robustness under renormalization group should be considered. We note that the Yukawa couplings have the specific structures in the hierarchical basis as

$$\widetilde{\Lambda}_u = \frac{\lambda}{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \widetilde{\Lambda}_{\mathcal{D}} = \frac{\lambda}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{15} \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & \sqrt{10}+* \end{pmatrix}.$$

Here, "$*$" denotes the dominant terms with the large USY phases for m_D's and the CP violation, while "0" the perturbation ones with the small USY phases for the ordinary quark masses, except for m_t, and mixings. By setting the small USY phases to be zero, chiral symmetries $\mathrm{U}(2)_{q\mathrm{L}} \times \mathrm{U}(2)_{u\mathrm{R}} \times \mathrm{U}(3)_{d\mathrm{R}}$ really appear. In particular, $\mathrm{U}(2)_{q\mathrm{L}}$ may break as $\mathrm{U}(2)_{q\mathrm{L}} \to \mathrm{U}(1)_{q1\mathrm{L}} \to$ non in accordance with Eq. (4.6). By virtue of these approximate symmetries the above USY structure is almost maintained under the renormalization group evolution. Then, by including the renormalization group corrections, the suitable USY phase values will be found at the unification scale in some reasonable range to reproduce the quark masses and CKM matrix with sufficient CP violation, as investigated so far.

In conclusion, we have investigated the quark masses and mixings in the USY scheme by including vector-like down-type quark singlets. In contrast with the standard model with USY, the sufficient CP violation is obtained for the CKM matrix through the mixing between the ordinary down-type quarks and quark singlets. Two or more quark singlets are needed to have the relevant large USY phases for the desired CP violation. These quark singlets may have masses $\sim$ TeV, to be discovered in the future collider experiments [56]. We have shown that with rather flexible choices of the USY phase values the actual quark masses and CKM matrix are really reproduced. Then, it is interesting for further investigations to invoke some textures and flavor symmetries for the USY phases so as to derive some predictive relations among the quark masses and mixings. The top-bottom hierarchy $m_t \gg m_b$ also appears naturally in the USY scheme in the presence of extra down-type quark singlets but no extra up-type quark singlets. Furthermore, in the USY scheme (or more generally flavor democracy), the fermion mass hierarchy may be extended as $m_t \gg m_b \sim m_\tau$ if vector-like lepton doublets are also present. In E_6-type models, such down-type quark singlets and lepton doublets are indeed accommodated in the **27** representation.

Chapter 5

Flavor-changing interactions with singlet quarks and their implications for the LHC

In this chapter, we investigate the flavor-changing interactions in an extension of the standard model with singlet quarks and singlet Higgs, which are induced by the mixing between the ordinary quarks and the singlet quarks (q-Q mixing). We consider the effects of the gauge and scalar interactions in the $\Delta F = 2$ mixings of K^0, B_d, B_s and D^0 mesons to show the currently allowed range of the q-Q mixing. Then, we investigate the new physics around the electroweak scale to the TeV scale, which is accessible to the Large Hadron Collider. Especially, the scalar coupling mediated by the singlet Higgs may provide distinct signatures for the decays of the singlet quarks and Higgs particles, which should be compared with the conventionally expected ones via the gauge and standard Higgs couplings. Observations of the singlet quarks and Higgs particles will present us important insights on the q-Q mixing and Higgs mixing.

5.1 Introduction

As the standard model has been established in current experiments, the appearance of new physics now attracts growing interests especially in the light of the Large Hadron Collider (LHC). So far various extensions of the standard model with their own motivations have been investigated for new physics, including exotic fermions, extra Higgs fields, extended gauge interactions, supersymmetry, and so on. The new physics might already provide some significant effects in the low-energy particle phenomena such as flavor-changing processes. It is now expected seriously that the new physics will reveal itself in the LHC experiments.

Among many intriguing extensions of the standard model, we here investigate the new physics provided by isosinglet quarks, which are suggested in certain models such as E_6-type unification [2, 3, 4]. Specifically, there are two types of singlet quarks, U with electric charge $Q_{\rm em} = 2/3$ and D with $Q_{\rm em} = -1/3$, which may mix with the ordinary quarks. It is also reasonable to incorporate a singlet Higgs field S, which provides the singlet quark masses and the q-Q mixing between the ordinary quarks ($q = u, d$) and the singlet quarks ($Q = U, D$). In this sort of model various novel features arise through the q-Q mixing [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 23, 24, 25, 26, 27, 38, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74]. The unitarity of Cabibbo-Kobayashi-Maskawa (CKM) matrix within the ordinary quark sector is violated, and the flavor-changing neutral currents (FCNC's) appear at the tree-level. These flavor-changing interactions are described appropriately in terms of the q-Q mixing parameters and quark masses [5, 6, 7, 8, 9, 23, 26]. Then, the actual CKM mixing is reproduced up to the small unitarity violation provided the FCNC's are suppressed sufficiently with the small q-Q mixing. In this respect the presence of singlet quarks may introduce an interesting extension of the notion of natural flavor conservation [29, 30, 75, 76]. Furthermore, the new CP-violating phases in q-Q mixing may provide significant contributions especially in the B meson physics [5, 6, 7, 8, 10, 11, 13, 14, 16, 24, 25, 38, 60, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74].

It is also expected in cosmology that the singlet quarks and singlet Higgs field may play important roles in the early universe. Specifically, in the first-order electroweak phase transition the CP-violating q-Q mixing via the coupling with the complex singlet Higgs S can be efficient to produce the chiral charge fluxes through the bubble wall for the baryogenesis [41, 42, 43]. Furthermore, the presence of singlet Higgs field is preferable for realizing the strong enough first-order electroweak phase transition.

As mentioned in the above, the singlet quarks and singlet Higgs bring various intriguing features in particle physics and cosmology. It is hence worth considering their phenomenological implications toward the discovery of them at the LHC [77, 78, 79, 80, 81, 82, 83]. In this chapter we investigate the flavor-changing interactions in the presence of singlet quarks and singlet Higgs. The effects of the gauge interactions have been investigated extensively in the literature [5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74]. Here, we rather note that the scalar interactions mediated by the singlet Higgs may provide significant effects in some cases [10, 17, 24, 25, 26, 38, 61, 62], which has not been considered thoroughly so far in the models with singlet quarks.

The rest of this chapter is organized as follows. In Sec. 5.2 we describe a representative model with singlet quarks and one complex singlet Higgs field, and review the essential features on the quark mixings and flavor-changing interactions. In Sec. 5.3 we consider the effects of the flavor-changing interactions in the $\Delta F = 2$ mixings of K^0, B_d, B_s and D^0 mesons to show the currently allowed range of the q-Q mixing. In Sec. 5.4 we investigate the decays of the singlet quarks and Higgs particles, which may provide distinct signatures upon their productions at the LHC. Sec. 5.5 is devoted to summary. In Appendix B a detailed derivation is presented for the suitable relations among the gauge and scalar couplings.

5.2 Quark mixings and flavor-changing interactions

We first review the the essential features on the quark mixings and flavor-changing interactions including the singlet quarks and singlet Higgs, which are described appropriately in terms of the q-Q mixing parameters and quark masses (see Chapter 3 and Ref. [26] for the detailed description). We consider a representative electroweak model based on the gauge symmetry $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, where singlet quarks U and D together with one complex singlet Higgs field S are incorporated. The generic Yukawa couplings are given by

$$\begin{aligned}
\mathcal{L}_{\mathrm{Y}} \;=\; & - u_0^c \lambda_u \Psi_{q_0} \Phi_H - U_0^c h_u \Psi_{q_0} \Phi_H \\
& - u_0^c (f_U S + f_U' S^\dagger) U_0 - U_0^c (\lambda_U S + \lambda_U' S^\dagger) U_0 \\
& - d_0^c \lambda_d V_0^\dagger \Psi_{q_0} \tilde{\Phi}_H - D_0^c h_d V_0^\dagger \Psi_{q_0} \tilde{\Phi}_H \\
& - d_0^c (f_D S + f_D' S^\dagger) D_0 - D_0^c (\lambda_D S + \lambda_D' S^\dagger) D_0 \\
& + \mathrm{H.c.}
\end{aligned} \tag{5.1}$$

in terms of the two-component Weyl fields for the electroweak eigenstates with subscript "0". (The generation indices and Lorentz factors are omitted here for simplicity.) The isodoublets of left-handed ordinary quarks are represented by

$$\Psi_{q_0} = \begin{pmatrix} u_0 \\ V_0 d_0 \end{pmatrix} \tag{5.2}$$

with a certain 3×3 unitary matrix V_0. The Higgs doublet is also given by

$$\Phi_H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{5.3}$$

with $\tilde{\Phi}_H \equiv i\tau_2 \Phi_H^*$. The Higgs fields develop vacuum expectation values (VEV's),

$$\langle H^0 \rangle = v/\sqrt{2},\ \langle S \rangle = v_S/\sqrt{2}, \tag{5.4}$$

where $v = (\sqrt{2}G_F)^{-1/2} = 246\mathrm{GeV}$, and $v_S \sim 100\mathrm{GeV} - 1\mathrm{TeV}$ is assumed. The possible complex phase δ_S of $\langle S \rangle$ is not presented explicitly for simplicity of notation, which may be effectively included in the Yukawa couplings and the Higgs potential terms at the tree level.

5.2.1 Quark masses and mixings

The quark mass matrix is produced as

$$\mathcal{M}_{\mathcal{Q}} = \begin{pmatrix} M_q & \Delta_{qQ} \\ \Delta'_{qQ} & M_Q \end{pmatrix}. \tag{5.5}$$

The submatrices are given by

$$\begin{aligned} M_q &= \lambda_q v/\sqrt{2}, \Delta'_{qQ} = h_q v/\sqrt{2}, & (5.6) \\ \Delta_{qQ} &= f_Q^+ v_S/\sqrt{2}, M_Q = \lambda_Q^+ v_S/\sqrt{2}, & (5.7) \end{aligned}$$

where

$$\begin{aligned} f_Q^+ &\equiv f_Q + f'_Q, f_Q^- \equiv i(f_Q - f'_Q), & (5.8) \\ \lambda_Q^+ &\equiv \lambda_Q + \lambda'_Q, \lambda_Q^- \equiv i(\lambda_Q - \lambda'_Q). & (5.9) \end{aligned}$$

Henceforth $\mathcal{Q} = (q, Q)$ collectively, and N_Q denotes the number of singlet quarks. The quark mass matrix $\mathcal{M}_{\mathcal{Q}}$ is diagonalized by unitary transformations $\mathcal{V}_{\mathcal{Q}_\mathrm{L}}$ and $\mathcal{V}_{\mathcal{Q}_\mathrm{R}}$ as

$$\mathcal{V}_{\mathcal{Q}_\mathrm{R}}^\dagger \mathcal{M}_{\mathcal{Q}} \mathcal{V}_{\mathcal{Q}_\mathrm{L}} = \bar{\mathcal{M}}_{\mathcal{Q}} = \begin{pmatrix} \bar{M}_q & \mathbf{0} \\ \mathbf{0} & \bar{M}_Q \end{pmatrix}, \tag{5.10}$$

where $\bar{M}_q = \mathrm{diag}(m_{q_1}, m_{q_2}, m_{q_3})$, $\bar{M}_Q = \mathrm{diag}(m_{Q_1}, \ldots)$, and $(q_1, q_2, q_3) = (u, c, t)$ or (d, s, b). The quark mass eigenstates q_i ($i = 1, 2, 3$) and Q_a ($a = 1, 2, \ldots, N_Q$) are given by

$$\begin{pmatrix} q \\ Q \end{pmatrix} = \mathcal{V}_{\mathcal{Q}_\mathrm{L}}^\dagger \begin{pmatrix} q_0 \\ Q_0 \end{pmatrix}, (q^c, Q^c) = (q_0^c, Q_0^c)\mathcal{V}_{\mathcal{Q}_\mathrm{R}} \tag{5.11}$$

with the $(3+N_Q)\times(3+N_Q)$ unitary matrices as

$$\mathcal{V}_{Q_\chi} = \begin{pmatrix} V_{q_\chi} & \epsilon_{q_\chi} \\ -\epsilon'^{\dagger}_{q_\chi} & V_{Q_\chi} \end{pmatrix} (\chi = \mathrm{L}, \mathrm{R}), \tag{5.12}$$

where ϵ_{q_χ} and ϵ'_{q_χ} represent the q-Q mixing.

The quark mass matrix $\mathcal{M}_{\mathcal{Q}}$ may be reduced to a specific form with either $\Delta'_{qQ} = \mathbf{0}$ or $\Delta_{qQ} = \mathbf{0}$ by a unitary transformation of the right-handed quarks. Then, the Yukawa coupling λ_q is diagonalized by unitary transformations of the ordinary quarks as

$$\lambda_q = \mathrm{diag}(\lambda_{q_1}, \lambda_{q_2}, \lambda_{q_3}), \tag{5.13}$$

while the condition $\Delta'_{qQ} = \mathbf{0}$ or $\Delta_{qQ} = \mathbf{0}$ is maintained. These transformations to specify the form of $\mathcal{M}_{\mathcal{Q}}$ do not mix the electroweak doublets with the singlets, respecting the $\mathrm{SU(3)}_C \times \mathrm{SU(2)}_W \times \mathrm{U(1)}_Y$. Hence, without loss of generality we may start with either of these bases, as seen in Chapter 3,

$$\textbf{basis (a)} : \Delta'_{qQ} = \mathbf{0}, \textbf{basis (b)} : \Delta_{qQ} = \mathbf{0}.$$

The Yukawa couplings h_q, f_Q, f'_Q, λ_Q, λ'_Q and the mixing matrix V_0 are redefined according to the transformations to specify the quark basis. In particular, $h_q = \mathbf{0}$ solely in the basis (a). On the other hand, in the basis (b) a specific relation $f'_Q = -f_Q$ ($f^+_Q = \mathbf{0}$) holds apparently though no tuning is imposed among the couplings in the original basis.

The q-Q mixings are given specifically in the basis (a) as

$$(\epsilon_{q_\mathrm{L}})_{ia} \sim (\epsilon'_{q_\mathrm{L}})_{ia} \sim (m_{q_i}/m_Q)\epsilon^f_i, \tag{5.14}$$

$$(\epsilon_{q_\mathrm{R}})_{ia} \sim (\epsilon'_{q_\mathrm{R}})_{ia} \sim \epsilon^f_i \tag{5.15}$$

in terms of the q-Q mixing parameters from the f^+_Q coupling,

$$\epsilon^f_i = (v_S/m_Q)\overline{|(f^+_Q)_{ia}|}/\sqrt{2} = \overline{|(\Delta_{qQ})_{ia}|}/m_Q, \tag{5.16}$$

where $m_Q = \overline{m_{Q_a}} \sim (\overline{|(f^+_Q)_{ia}|} + \overline{|(\lambda^+_Q)_{ab}|})v_S$, and the bar denotes the mean value. The left-handed q-Q mixing is suppressed significantly by the q/Q mass ratios m_{q_i}/m_Q [5, 6, 7, 8, 9, 23, 26]. On the other hand, in the basis (b)

$$(\epsilon_{q_\mathrm{L}})_{ia} \sim (\epsilon'_{q_\mathrm{L}})_{ia} \sim \epsilon^h_i, \tag{5.17}$$

$$(\epsilon_{q_\mathrm{R}})_{ia} \sim (\epsilon'_{q_\mathrm{R}})_{ia} \sim (m_{q_i}/m_Q)\epsilon^h_i \tag{5.18}$$

in terms of the q-Q mixing parameters from the h_q coupling,

$$\epsilon_i^h = (v/m_Q)\overline{|(h_q)_{ai}|}/\sqrt{2} = \overline{|(\Delta'_{qQ})_{ai}|}/m_Q. \tag{5.19}$$

The left-handed q-Q mixing is no longer suppressed by the q/Q mass ratios.

We may move from the basis (a) with $\Delta'_{qQ} = \mathbf{0}$ to the basis (b) with $\Delta_{qQ} = \mathbf{0}$ by using a unitary transformation. Here, the left-handed q-Q mixings in the bases (a) and (b) are related as $\epsilon_i^h \sim (m_{q_i}/m_Q)\epsilon_i^f$ so that Eq. (5.14) is apparently reproduced from Eq. (5.17). Hence, the basis (a) may be regarded as a special case of the basis (b) [26]. If $\overline{|f_Q|} + \overline{|f'_Q|} \gtrsim \overline{|\lambda_Q|} + \overline{|\lambda'_Q|}$ providing $\epsilon_i^f \sim 1$ in the basis (a), then it is suitable to adopt the basis (b) alternatively. The see-saw basis with $M_q = \mathbf{0}$ is also possible for $N_Q = 3$ [31, 32, 33, 34]. Since it is related to the bases (a) and (b) having a hybrid feature for the quark mixing [26], we do not consider explicitly the see-saw model. We adopt complementarily the bases (a) and (b), where the ordinary quark masses are reproduced as

$$m_{q_i} = c_i \lambda_{q_i} v/\sqrt{2} \tag{5.20}$$

with $c_i \sim 1$ depending on the small q-Q mixing [5, 6, 7, 8, 9, 23, 26]. In the general basis with $\Delta_{qQ} \neq \mathbf{0}$ and $\Delta'_{qQ} \neq \mathbf{0}$, e.g., the see-saw model, the ordinary quark mass hierarchy is not described clearly in terms of the Yukawa couplings λ_{q_i}.

5.2.2 Flavor-changing interactions

The CKM matrix V for the W-boson coupling with the ordinary quarks is given by

$$V = V_{u_{\rm L}}^\dagger V_0 V_{d_{\rm L}}, \tag{5.21}$$

where $V_{u_{\rm L}}$ and $V_{d_{\rm L}}$ are the 3×3 submatrices in Eq. (5.12). The unitarity violation of V is induced at the second order of q-Q mixing with $\epsilon_{q_{\rm L}}\epsilon_{q_{\rm L}}^\dagger$ and $\epsilon'_{q_{\rm L}}\epsilon'^\dagger_{q_{\rm L}}$, which should be suppressed enough phenomenologically [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 23, 24, 25, 26, 27, 38, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74]. Then, the realistic CKM matrix V is reproduced by taking suitably the original V_0.

The modification of the left-handed Z-boson coupling with the ordinary quarks is also given at the second order of q-Q mixing [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 23, 24, 25, 26, 27, 38, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74] as

$$\Delta \mathcal{Z}_{\mathcal{Q}}[q^\dagger q] = -\epsilon'_{q_{\rm L}}\epsilon'^\dagger_{q_{\rm L}}, \tag{5.22}$$

where the Z-boson coupling is presented by removing the isospin factor $I_3(q_0)$ with $I_3(u_0) = 1/2$ and $I_3(d_0) = -1/2$ for simplicity of notation. The right-handed coupling is, on the other hand, unchanged as $\Delta\mathcal{Z}_{\mathcal{Q}^c} = \mathbf{0}$ for $I_3(q_0^c) = I_3(Q_0^c) = 0$. Specifically, in the basis (a) we have

$$\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ij}(\mathrm{a}) \sim (m_{q_i}/m_Q)(m_{q_j}/m_Q)\epsilon_i^f\epsilon_j^f. \tag{5.23}$$

This correction as well as the CKM unitarity violation are suppressed substantially by the second order of q/Q mass ratios. Alternatively, in the basis (b) we have

$$\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ij}(\mathrm{b}) \sim \epsilon_i^h\epsilon_j^h, \tag{5.24}$$

which is no longer suppressed by the q/Q mass ratios. Then, significant constraints are placed phenomenologically on the q-Q mixings $\epsilon_i^h \ll 1$, which have been investigated extensively in the literature [11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 60, 61, 62, 63, 64, 65, 66].

The neutral scalar couplings of the quarks $\mathcal{Q} = \mathcal{U}, \mathcal{D}$ are extracted from Eq. (5.1) as

$$\mathcal{L}_\phi(\mathcal{Q}) = -\sum_{\phi_r^0 = H, S_+, S_-} \mathcal{Q}^c\Lambda_{\mathcal{Q}}^{\phi_r^0}\mathcal{Q}\phi_r^0 + \mathrm{h.c.}, \tag{5.25}$$

where

$$\Lambda_{\mathcal{Q}}^H = \frac{1}{\sqrt{2}}\mathcal{V}_{\mathcal{Q}_\mathrm{R}}^\dagger \begin{pmatrix} \lambda_q & \mathbf{0} \\ h_q & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_\mathrm{L}}, \tag{5.26}$$

$$\Lambda_{\mathcal{Q}}^{S_\pm} = \frac{1}{\sqrt{2}}\mathcal{V}_{\mathcal{Q}_\mathrm{R}}^\dagger \begin{pmatrix} \mathbf{0} & f_Q^\pm \\ \mathbf{0} & \lambda_Q^\pm \end{pmatrix} \mathcal{V}_{\mathcal{Q}_\mathrm{L}}. \tag{5.27}$$

The real neutral scalar fields, $\phi_1^0 = H \equiv \sqrt{2}\mathrm{Re}(H^0 - \langle H^0\rangle)$, $\phi_2^0 = S_+ \equiv \sqrt{2}\mathrm{Re}(S - \langle S\rangle)$, $\phi_3^0 = S_- \equiv \sqrt{2}\mathrm{Im}(S - \langle S\rangle)$, mix generally to form the mass eigenstates ϕ_r ($r = 1, 2, 3$) through an orthogonal transformation O_ϕ:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = O_\phi \begin{pmatrix} H \\ S_+ \\ S_- \end{pmatrix}. \tag{5.28}$$

The Nambu-Goldstone mode $G \equiv \sqrt{2}\mathrm{Im}(H^0 - \langle H^0\rangle)$ is absorbed by the Z boson.

The submatrices $\Lambda_{\mathcal{Q}}^{\phi_r^0}[q^c q]$ of these neutral scalar couplings for the ordinary quarks are given by

$$\Lambda_{\mathcal{Q}}^H[q^c q] = V_{q_\mathrm{R}}^\dagger\lambda_q V_{q_\mathrm{L}} - \epsilon'_{q_\mathrm{R}} h_q V_{q_\mathrm{L}}, \tag{5.29}$$

$$\Lambda_{\mathcal{Q}}^{S_\pm}[q^c q] = -V_{q_\mathrm{R}}^\dagger f_Q^\pm \epsilon'^\dagger_{q_\mathrm{L}} + \epsilon'_{q_\mathrm{R}}\lambda_Q^\pm\epsilon'^\dagger_{q_\mathrm{L}}. \tag{5.30}$$

Here, some close relations hold for the gauge and scalar couplings (see the Appendix B for derivation). The coupling of the standard Higgs H is given actually as

$$\Lambda^H_{\mathcal{Q}}[q^c q]_{ij} \;=\; (m_{q_i}/v)(\delta_{ij} + \Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ij}) \tag{5.31}$$

with the q-Q mixing induced Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$ in Eq. (5.22). Similarly, the coupling of the singlet Higgs S_+ is calculated as

$$\Lambda^{S_+}_{\mathcal{Q}}[q^c q]_{ij} = -(m_{q_i}/v_S)\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ij}. \tag{5.32}$$

Hence the q-Q mixing effects for the scalar couplings $\Lambda^H_{\mathcal{Q}}[q^c q]$ and $\Lambda^{S_+}_{\mathcal{Q}}[q^c q]$ are always sub-leading compared with those for the Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$, which is due to the suppression by m_{q_i}/v from the chirality flip. It is rather remarkable that the coupling of the singlet Higgs S_- may be dominant without a close relation to the Z-boson coupling. In the basis (a) we have

$$\Lambda^{S_-}_{\mathcal{Q}}[q^c q]_{ij}(\mathrm{a}) \sim (m_{q_j}/v_S)\epsilon^f_i \epsilon^f_j, \tag{5.33}$$

where $(\epsilon'^\dagger_{q_{\rm L}})_{aj} \sim (m_{q_j}/m_Q)\epsilon^f_j$, $(\epsilon'_{q_{\rm R}})_{ia} \sim \epsilon^f_i$, $(\lambda^-_Q)_{ab} \sim (m_Q/v_S)$ and $(f^-_Q)_{ia} \sim (m_Q/v_S)\epsilon^f_i$ are applied in Eq. (5.30). In contrast to the Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$ in Eq. (5.23), the scalar coupling $\Lambda^{S_-}_{\mathcal{Q}}[q^c q]$ in Eq. (5.33) is suppressed only by the first order of ordinary quark mass. In the basis (b), by applying $(\epsilon'^\dagger_{q_{\rm L}})_{aj} \sim \epsilon^h_j$ and $(f^-_Q)_{ia} \sim (m_Q/v_S)\epsilon^f_i$ in Eq. (5.30), we estimate

$$\Lambda^{S_-}_{\mathcal{Q}}[q^c q]_{ij}(\mathrm{b}) \;\sim\; (m_Q/v_S)\epsilon^f_i \epsilon^h_j, \tag{5.34}$$

up to the sub-leading contribution of the second term $\sim (m_{q_i}/v_S)\epsilon^h_i\epsilon^h_j$ in Eq. (5.30). Here, similar to Eq. (5.16), the q-Q mixing parameters are introduced for convenience as $\epsilon^f_i = (v_S/m_Q)\overline{|2(f_Q)_{ia}|}/\sqrt{2}$ even though $f^+_Q = \mathbf{0}$ ($f^-_Q = 2if_Q$ with $f_Q = -f'_Q$) for $\Delta_{qQ} = \mathbf{0}$ in the basis (b). It should also be noted, as discussed previously, that by considering the relation for the left-handed q-Q mixing,

$$\epsilon^h_i \sim (m_{q_i}/m_Q)\epsilon^f_i, \tag{5.35}$$

Eqs. (5.24) and (5.34) in the basis (b) reproduce Eqs. (5.23) and (5.33) in the basis (a), respectively.

We mention for completeness that in the case of one real S (or one supersymmetric S) with the f_Q and λ_Q couplings ($f'_Q \equiv \mathbf{0}$ and $\lambda'_Q \equiv \mathbf{0}$), the scalar coupling $\Lambda^S_{\mathcal{Q}}[q^c q]$ is given by Eq. (5.32) for $\Lambda_{\mathcal{Q}}^{S+}[q^c q]$ related to the Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$. On the other hand, if the bare mass term M_Q is adopted instead of the λ_Q coupling while the f_Q coupling provides the q-Q mixing, the scalar coupling $\Lambda_{\mathcal{Q}}^{S}[q^c q]$ is rather given by Eqs. (5.33) and (5.34) for $\Lambda_{\mathcal{Q}}^{S-}[q^c q]$ even in the case of one real S.

5.3 Singlet quark Effects in $\Delta F = 2$ mixings of neutral mesons

We perform a detailed analysis on the q-Q mixing effects in the $\Delta F = 2$ mixings of K^0, B_d, B_s and D^0 mesons, by considering the general bounds for new physics which are presented in Ref. [84]. The Z-mediated FCNC's in $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$ have been investigated extensively in the literature [5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74]. By placing experimental constraints on the left-handed q-Q mixings $(\epsilon_{q_{\rm L}})_{ia} \sim (\epsilon'_{q_{\rm L}})_{ia} \sim \epsilon_i^h$ in the basis (b), these analyses have discussed the possibility of new physics provided by the singlet quarks, in particular, for the B meson physics. Here, we rather note that in some cases the scalar FCNC's in $\Lambda_{\mathcal{Q}}^{S-}[q^c q]$ may dominate over the Z-mediated FCNC's in $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$, providing distinct signals for new physics [10, 17, 24, 25, 26, 38, 61, 62]. This intriguing possibility has not been paid so much attention so far in the models with singlet quarks.

The effective Hamiltonian contributing to the $\Delta F = 2$ mixing of the neutral meson M (K^0, B_d, B_s, D^0) is given generally [84] as

$$\mathcal{H}_{\rm eff}^{\Delta F=2} = \sum_{k=1}^{5} C_M^k \mathcal{O}_k^{q_i q_j} + \sum_{k=1}^{3} \tilde{C}_M^k \tilde{\mathcal{O}}_k^{q_i q_j}, \tag{5.36}$$

where

$$q_i q_j = sd(K^0), bd(B_d), bs(B_s), cu(D^0). \tag{5.37}$$

The four-quark operators are

$$\begin{aligned}
\mathcal{O}_1^{q_i q_j} &= \bar{q}_{j\rm L}^\alpha \gamma_\mu q_{i\rm L}^\alpha \bar{q}_{j\rm L}^\beta \gamma^\mu q_{i\rm L}^\beta, \mathcal{O}_2^{q_i q_j} = \bar{q}_{j\rm R}^\alpha q_{i\rm L}^\alpha \bar{q}_{j\rm R}^\beta q_{i\rm L}^\beta, \\
\mathcal{O}_3^{q_i q_j} &= \bar{q}_{j\rm R}^\alpha q_{i\rm L}^\beta \bar{q}_{j\rm R}^\beta q_{i\rm L}^\alpha, \mathcal{O}_4^{q_i q_j} = \bar{q}_{j\rm R}^\alpha q_{i\rm L}^\alpha \bar{q}_{j\rm L}^\beta q_{i\rm R}^\beta, \\
\mathcal{O}_5^{q_i q_j} &= \bar{q}_{j\rm R}^\alpha q_{i\rm L}^\beta \bar{q}_{j\rm L}^\beta q_{i\rm R}^\alpha,
\end{aligned}$$

and α and β denote the colors. The operators $\tilde{\mathcal{O}}_{1,2,3}^{q_i q_j}$ are obtained from the operators $\mathcal{O}_{1,2,3}^{q_i q_j}$ by the exchange L $\leftrightarrow$ R. The coefficients in the effective Hamiltonian at the scale

$\mu = m_Q$ of singlet quarks are calculated as

$$
\begin{aligned}
C_M^1(m_Q) &= (g/2\cos\theta_W)^2(\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ji})^2/m_Z^2, && (5.38)\\
C_M^2(m_Q) &= (\Lambda_{\mathcal{Q}}^{S_-}[q^c q]_{ji})^2/m_{S_-}^2, && (5.39)\\
\tilde{C}_M^2(m_Q) &= (\Lambda_{\mathcal{Q}}^{S_-}[q^c q]_{ij}^*)^2/m_{S_-}^2, && (5.40)\\
C_M^4(m_Q) &= (\Lambda_{\mathcal{Q}}^{S_-}[q^c q]_{ji})(\Lambda_{\mathcal{Q}}^{S_-}[q^c q]_{ij}^*)/m_{S_-}^2, && (5.41)
\end{aligned}
$$

and the others are zero. Here, the Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$ and the dominant scalar coupling $\Lambda_{\mathcal{Q}}^{S_-}[q^c q]$ are considered, and the scalar mixing in O_ϕ is neglected for simplicity. By requiring that these coefficients in Eqs. (5.38) – (5.41) are all within the bounds presented specifically in Table 4 of Ref. [84], we find the allowed range of the q-Q mixing depending on the masses $m_Q, m_{S_-} \sim v_S$ of the singlet quarks Q and singlet Higgs S_-.

The constraints on the q-Q mixing are given roughly below, where $m_D = m_U = v_S =$ 500GeV and $m_{S_-} = 0.6 v_S = 300$GeV are taken typically to estimate the FCNC's with Eqs. (5.23), (5.24), (5.33) and (5.34) in terms of the q-Q mixing parameters ϵ_i^f and ϵ_i^h. In the basis (a) significant constraints on the d-D mixing are placed by the scalar coupling $\Lambda_{\mathcal{D}}^{S_-}[d^c d]$ for C_M^2, $\tilde{C}_M^2$ and C_M^4 ($M = K^0, B_d, B_d$) as

$$
\Lambda_{\mathcal{D}}^{S_-}(\mathrm{a}):\ \begin{array}{ll}
(\epsilon_1^f\epsilon_2^f)^{\frac{1}{2}} \lesssim 0.1/\delta_{12}^{\frac{1}{4}} & (\mathrm{Im}K)\\
(\epsilon_1^f\epsilon_2^f)^{\frac{1}{2}} \lesssim 0.4 & (\mathrm{Re}K)\\
(\epsilon_1^f\epsilon_3^f)^{\frac{1}{2}} \lesssim 0.2 & (|B_d|)\\
(\epsilon_2^f\epsilon_3^f)^{\frac{1}{2}} \lesssim 0.5 & (|B_s|).
\end{array} \qquad (5.42)
$$

Here, the effective CP-violating phases in the FCNC's contributing to the K^0-$\bar{K}^0$ mixing are denoted collectively by δ_{12}. No significant constraints are, on the other hand, placed by the Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{D}}[d^\dagger d]$ which is substantially suppressed by the second order of d/D mass ratios in Eq. (5.23). Alternatively, in the basis (b) constraints on the d-D mixing are given as

$$
\Lambda_{\mathcal{D}}^{S_-}(\mathrm{b}):\ \begin{array}{ll}
(\epsilon_1^f\epsilon_2^h)^{\frac{1}{2}}, (\epsilon_2^f\epsilon_1^h)^{\frac{1}{2}} \lesssim 1\times 10^{-3}/\delta_{12}^{\frac{1}{4}} & (\mathrm{Im}K)\\
(\epsilon_1^f\epsilon_2^h)^{\frac{1}{2}}, (\epsilon_2^f\epsilon_1^h)^{\frac{1}{2}} \lesssim 4\times 10^{-3} & (\mathrm{Re}K)\\
(\epsilon_1^f\epsilon_3^h)^{\frac{1}{2}}, (\epsilon_3^f\epsilon_1^h)^{\frac{1}{2}} \lesssim 0.01 & (|B_d|)\\
(\epsilon_2^f\epsilon_3^h)^{\frac{1}{2}}, (\epsilon_3^f\epsilon_2^h)^{\frac{1}{2}} \lesssim 0.03 & (|B_s|),
\end{array} \qquad (5.43)
$$

$$
\Delta\mathcal{Z}_{\mathcal{D}}(\mathrm{b}):\ \begin{array}{ll}
(\epsilon_1^h\epsilon_2^h)^{\frac{1}{2}} \lesssim 4\times 10^{-3}/\delta_{12}^{\frac{1}{4}} & (\mathrm{Im}K)\\
(\epsilon_1^h\epsilon_2^h)^{\frac{1}{2}} \lesssim 0.02 & (\mathrm{Re}K)\\
(\epsilon_1^h\epsilon_3^h)^{\frac{1}{2}} \lesssim 0.03 & (|B_d|)\\
(\epsilon_2^h\epsilon_3^h)^{\frac{1}{2}} \lesssim 0.09 & (|B_s|).
\end{array} \qquad (5.44)
$$

Here, the constraints for the basis (a) in Eq. (5.42) are reproduced roughly from those for the basis (b) in Eq. (5.43) under the relation in Eq. (5.35). Constraints on the u-U mixing are estimated in the bases (a) and (b) as

$$\Lambda_{\mathcal{U}}^{S-}(\mathrm{a}): \quad (\epsilon_1^f \epsilon_2^f)^{\frac{1}{2}} \lesssim 0.2 \quad (|D^0|), \tag{5.45}$$

$$\Lambda_{\mathcal{U}}^{S-}(\mathrm{b}): \quad (\epsilon_1^f \epsilon_2^h)^{\frac{1}{2}}, (\epsilon_2^f \epsilon_1^h)^{\frac{1}{2}} \lesssim 8 \times 10^{-3} \quad (|D^0|), \tag{5.46}$$

$$\Delta \mathcal{Z}_{\mathcal{U}}(\mathrm{b}): \quad (\epsilon_1^h \epsilon_2^h)^{\frac{1}{2}} \lesssim 0.01 \quad (|D^0|). \tag{5.47}$$

Here, we comment on the contributions of the one-loop diagrams involving the singlet quarks and gauge bosons, which are enhanced by the factor $(m_Q/m_W)^2 g^2/(4\pi)^2$ compared with the tree-diagram contributions considered so far. They may dominate over the tree-diagram contributions for $m_Q > 1$TeV, though we are mainly interested in the singlet quarks below 1TeV in this analysis. The allowed range of the q-Q mixing will hence be reduced slightly for $m_Q \sim 1$TeV. However, for a larger $m_Q \sim 10$TeV, the loop contributions decrease as $(\epsilon_i^h \epsilon_j^h)^2 (m_Q/m_W)^2 \propto 1/m_Q^2$ with $\epsilon_i^h \propto 1/m_Q$ in Eq. (5.19). In such a case of $m_Q \sim 10$TeV, the gauge-mediated q-Q mixing effects to the $\Delta F = 2$ meson mixings are anyway below the current experimental bounds since the q-Q mixing becomes significantly small as $\epsilon_i^h < 0.02$ for $\overline{|(h_q)_{ai}|} < 1$ in Eq. (5.19).

In supplement to the above constraints on the q-Q mixing parameters from the FCNC's, it is also relevant to consider the constraints from the flavor-diagonal Z-boson couplings [9, 12, 14]. The observed branching ratios of the decays $Z \to q_i \bar{q}_i$ imply that the deviations of the flavor-diagonal Z-boson couplings from the standard model values should be small enough. Specifically, in the basis (b) with Eq. (5.24) constraints on the q-Q mixing may be placed roughly as

$$\Delta \mathcal{Z}_{\mathcal{Q}}(\mathrm{b}) \quad : \quad \epsilon_i^h \lesssim 0.03 \leftarrow |\Delta \mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ii}| \lesssim 10^{-3}. \tag{5.48}$$

On the other hand, in the basis (a) $\Delta \mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ii}$ of Eq. (5.23) is safely suppressed by $(m_{q_i}/m_Q)^2$ except for $q_i = t$.

To be more quantitative, we present the results of detailed numerical calculations for the d-D mixing effects in the down-type quark sector with one singlet D quark ($N_D = 1$ and $N_U = 0$) as a typical case. We take various values for the model parameters in a reasonable range as

$$\begin{aligned}
&v = 246\mathrm{GeV},\ v_S = 500\mathrm{GeV}, \\
&\lambda_{d_i} = \lambda_{d_i}^{(0)} = \sqrt{2} m_{d_i}/v \ \text{(preliminary)}, \\
&|\lambda_D|, |\lambda'_D| \in [0.3, 1.0] \sim m_D/v_S, \\
&(v/v_S)|(h_d)_i|/|\lambda_D^+| \in [0, 0.05] \sim \epsilon_i^h, \\
&|(f_D)_i|/|\lambda_D^+|, |(f'_D)_i|/|\lambda_D^+| \in [0, 2.0] \sim \epsilon_i^f, \\
&\arg[h_d, f_D, f'_D, \lambda_D, \lambda'_D] \in [-0.3\pi, 0.3\pi].
\end{aligned}$$

Here, the VEV v_S of the singlet Higgs S is fixed to a typical value for definiteness. The complex phases of the Yukawa couplings $h_d, f_D, f'_D, \lambda_D, \lambda'_D$ contribute to the CP violation in the FCNC's such as δ_{12} for ϵ_K of the K^0-$\bar{K}^0$ mixing. The total quark mass matrix $\mathcal{M}_{\mathcal{D}}$ (4×4 for $N_D = 1$) in Eq. (5.5) is given for a set of values of the model parameters. This preliminary $\mathcal{M}_{\mathcal{D}}$ with $\lambda_{d_i} = \lambda_{d_i}^{(0)}$ is diagonalized to evaluate the eigenvalues $m_{d_i}^{(0)}$ for the ordinary quark masses. Then, by considering the ratios $m_{d_i}^{(0)}/m_{d_i} \sim 1$ we adjust λ_{d_i} to obtain the actual quark masses m_{d_i}:

$$\lambda_{d_i} \to m_d = 5\mathrm{MeV},\ m_s = 110\mathrm{MeV},\ m_b = 4.2\mathrm{GeV}.$$

The singlet quark mass m_D is obtained for the above range of the model parameters as

$$m_D \sim 100\mathrm{GeV} - 1\mathrm{TeV}(v_S = 500\mathrm{GeV}).$$

At the same time, the unitary transformations $\mathcal{V}_{\mathcal{D}_\mathrm{L}}$ and $\mathcal{V}_{\mathcal{D}_\mathrm{R}}$ to specify the quark mass eigenstates are calculated. The actual CKM matrix V is reproduced by adjusting the original unitary matrix V_0 as $V_0 = V V_{d_\mathrm{L}}^{-1} \simeq V V_{d_\mathrm{L}}^\dagger$ ($V_{u_\mathrm{L}} = \mathbf{1}$ for $N_U = 0$):

$$V_0 \to V(\mathrm{CKM}).$$

As long as the q-Q mixing is small enough to satisfy the constraints from the $\Delta F = 2$ meson mixings, the unitarity violation of the CKM matrix is safely suppressed.

By using these results on the quark masses and q-Q mixings, we evaluate the couplings of the quarks with the gauge bosons and Higgs particles. Then, the contributions of the d-D mixing induced FCNC's to the effective Hamiltonian $\mathcal{H}_{\mathrm{eff}}^{\Delta F=2}$ for the K^0, B_d and B_s mixings are evaluated with Eqs. (5.38) – (5.41). They are compared with the experimental bounds presented in Table 4 of Ref. [84] to find the allowed range of the

d-D mixing parameters ϵ_i^f and ϵ_i^h. In this analysis, the masses of the Higgs particles are taken typically as

$$m_H = 120\text{GeV}, \ m_{S_+} = m_{S_-} = 300\text{GeV}. \tag{5.49}$$

Note here that the contributions of the S_- coupling in Eqs. (5.39) – (5.41) are proportional to $1/m_{S_-}^2$. Hence, as m_{S_-} is larger, the allowed range of the q-Q mixing parameters is extended further.

We have made the above calculations for many samples of the model parameter values. We show some characteristic results in the following. The portions of the d-D mixing effects in the $\Delta F = 2$ meson mixings for the experimental bounds $|C_M^k|_{\max}$ [84] are denoted by

$$r_k(M) \equiv |C_M^k|/|C_M^k|_{\max}. \tag{5.50}$$

In Fig. 5.1 scatter plots of $r_2(B_d)$ ($\circ$) and $r_2(B_s)$ ($\blacktriangle$) versus $(\epsilon_1^f\epsilon_3^f)^{1/2}$ are shown for the B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ mixings, respectively, which are provided by the singlet S_- coupling $\Lambda_{\mathcal{D}}^{S_-}[d^c d]$ in the basis (a). Similar plots of $r_2(B_s)$ ($\blacktriangle$) and $r_2(B_d)$ ($\circ$) versus $(\epsilon_2^f\epsilon_3^f)^{1/2}$ are shown in Fig. 5.2. The bounds for the K^0-$\bar{K}^0$ mixing have been checked to be satisfied in these plots. The results in Figs. 5.1 and 5.2 are in accordance with the rough estimates to obtain the bounds in Eq. (5.42). The dominant effects of the S_- coupling $\Lambda_{\mathcal{D}}^{S_-}[d^c d]$ of Eq. (5.33) are estimated in Eq. (5.39) as $|C_{B_d}^2| \sim (m_b/v_S)^2(\epsilon_1^f\epsilon_3^f)^2/m_{S_-}^2$ and $|C_{B_s}^2| \sim (m_b/v_S)^2(\epsilon_2^f\epsilon_3^f)^2/m_{S_-}^2$. In the log-log plot of Fig. 5.1, $r_2(B_d)$ ($\circ$) shows roughly the expected linear correlation with $(\epsilon_1^f\epsilon_3^f)^{\frac{1}{2}}$, while $r_2(B_s)$ ($\blacktriangle$) are distributed independently of $(\epsilon_1^f\epsilon_3^f)^{\frac{1}{2}}$. We see the similar feature in Fig. 5.2 for $r_2(B_s)$ ($\blacktriangle$) and $r_2(B_d)$ ($\circ$) versus $(\epsilon_2^f\epsilon_3^f)^{\frac{1}{2}}$. Precisely, in the basis (a) the d-D mixing parameters ϵ_i^f are defined with the f_D^+ coupling, while the singlet S_- coupling $\Lambda_{\mathcal{D}}^{S_-}$ is given by the f_D^- coupling ($|f_D^+| \sim |f_D^-|$ generally). This fact provides the appreciable spreads in the plots of $r_2(B_d)$ versus $(\epsilon_1^f\epsilon_3^f)^{\frac{1}{2}}$ and $r_2(B_s)$ versus $(\epsilon_2^f\epsilon_3^f)^{\frac{1}{2}}$. We find in these plots that the d-D mixing parameters $(\epsilon_1^f\epsilon_3^f)^{\frac{1}{2}}$ and $(\epsilon_2^f\epsilon_3^f)^{\frac{1}{2}}$ are really constrained for $r_2(B_d) \leq 1$ and $r_2(B_s) \leq 1$, respectively, as shown in Eq. (5.42). We note particularly that both the bounds for the B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ mixings may be saturated simultaneously with $\epsilon_3^f \sim 1$ and $(\epsilon_1^f\epsilon_2^f)^{\frac{1}{2}} \sim 0.1$ without conflicting with the bonds for the K^0-$\bar{K}^0$ mixing. Generally, in the basis (a) the right-handed d-D mixing is rather tolerable with $\epsilon_i^f \sim 0.1 - 1$. This is because the right-handed components of the ordinary and singlet quarks are indistinguishable with respect to the gauge interactions.

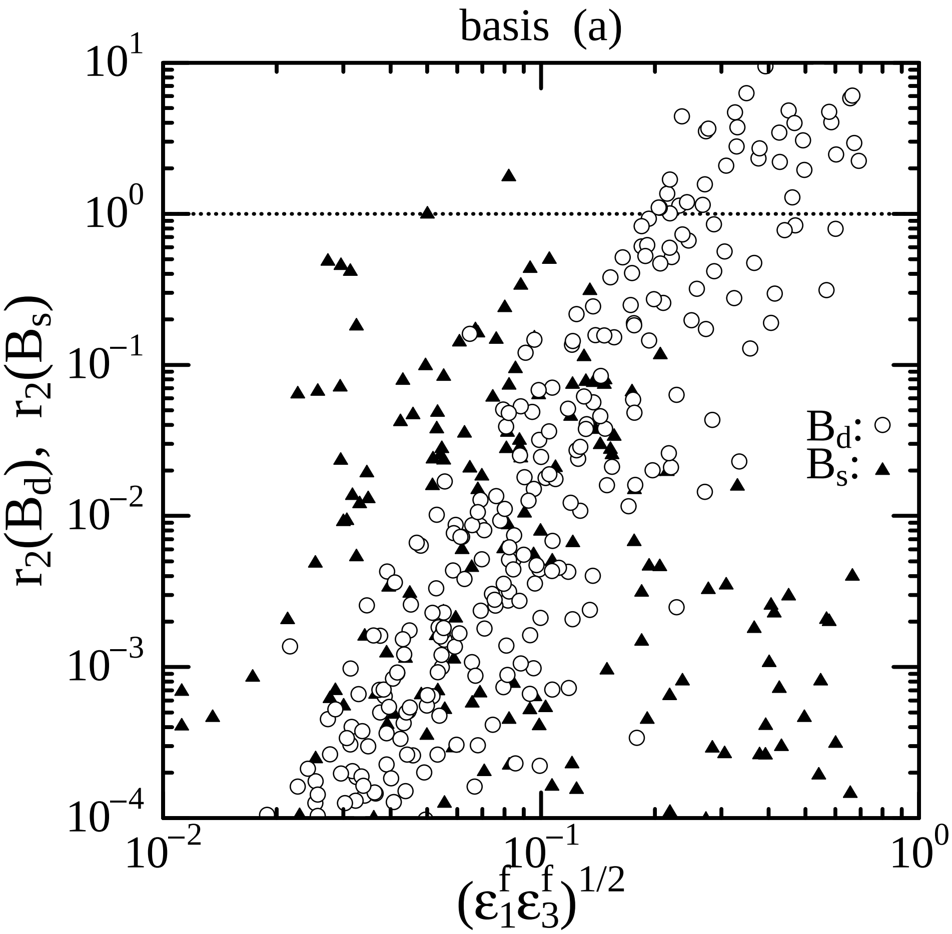

Figure 5.1: Scatter plots of $r_2(B_d) \equiv |C^2_{B_d}|/|C^2_{B_d}|_{\max}$ ($\circ$) and $r_2(B_s) \equiv |C^2_{B_s}|/|C^2_{B_s}|_{\max}$ ($\blacktriangle$) versus $(\epsilon_1^f\epsilon_3^f)^{1/2}$ for the B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ mixings, respectively, which are provided by the S_- coupling $\Lambda_{\mathcal{D}}^{S_-}[d^c d]$ in the basis (a).

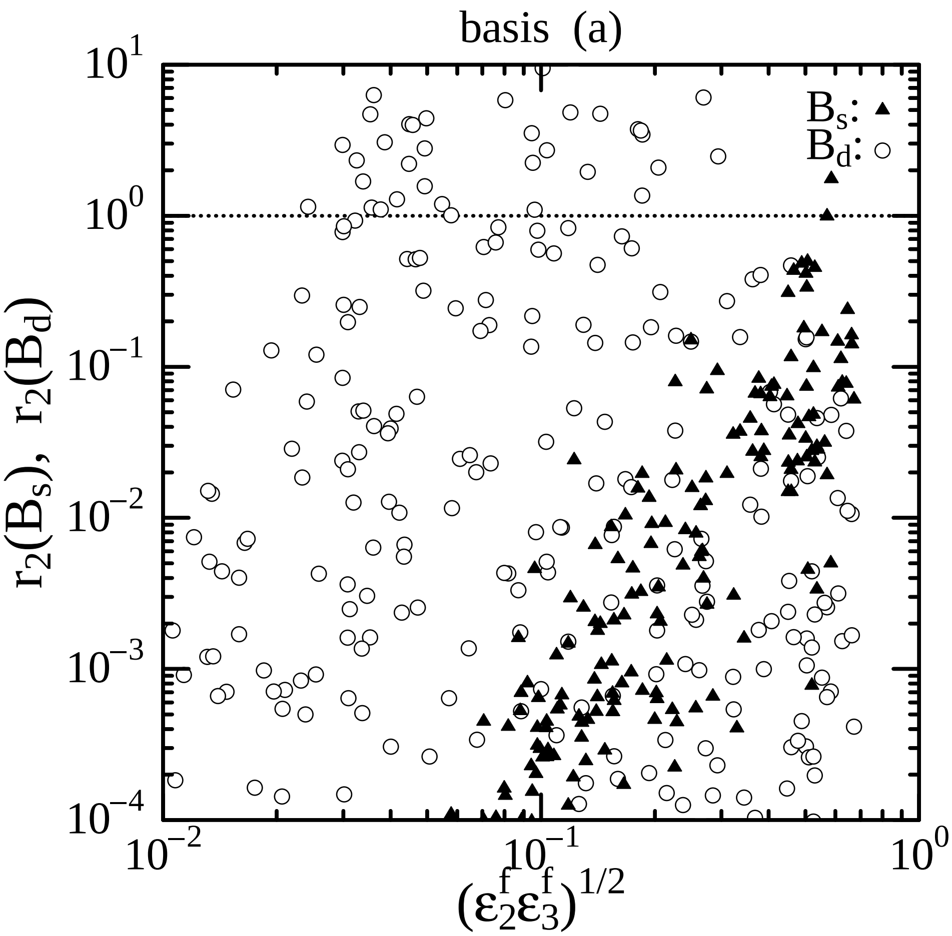

Figure 5.2: Scatter plots of $r_2(B_s) \equiv |C^2_{B_s}|/|C^2_{B_s}|_{\max}$ (▲) and $r_2(B_d) \equiv |C^2_{B_d}|/|C^2_{B_d}|_{\max}$ (○) versus $(\epsilon_2^f \epsilon_3^f)^{1/2}$, similarly to Fig. 5.1.

In Fig. 5.3 scatter plots of $r_1(B_d)$ ($\bullet$) and $r_2(B_d)$ ($\circ$) versus $(\epsilon_1^h \epsilon_3^h)^{1/2}$ are shown for the B_d-$\bar{B}_d$ mixing, which are provided by the Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{D}}[d^\dagger d]$ and the singlet S_- coupling $\Lambda_{\mathcal{D}}^{S_-}[d^c d]$, respectively, in the basis (b). Similar plots of $r_1(B_s)$ ($\triangle$) and $r_2(B_s)$ ($\blacktriangle$) versus $(\epsilon_2^h \epsilon_3^h)^{1/2}$ are shown in Fig. 5.4 for the B_s-$\bar{B}_s$ mixing. The bounds for the K^0-$\bar{K}^0$ mixing have been checked to be satisfied in these plots. The flavor-diagonal Z-boson couplings have also been checked to satisfy $|\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]_{ii}| < 3 \times 10^{-3}$, as considered in Eq. (5.48). The effects of the Z-boson coupling $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$ of Eq. (5.24) are estimated in Eq. (5.38) for the basis (b) as $|C_{B_d}^1| \sim (\epsilon_1^h \epsilon_3^h)^2 \sqrt{2} G_F$ and $|C_{B_s}^1| \sim (\epsilon_2^h \epsilon_3^h)^2 \sqrt{2} G_F$. In the log-log plot of Fig. 5.3, $r_1(B_d)$ ($\bullet$) shows clearly the linear correlation with $(\epsilon_1^h \epsilon_3^h)^{\frac{1}{2}}$ [here more precisely $(\epsilon_{q_{\rm L}})_i \simeq (\epsilon'_{q_{\rm L}})_i \simeq \epsilon_i^h$ for the left-handed q-Q mixing]. This is also the case in Fig. 5.4 for $r_1(B_s)$ ($\triangle$) versus $(\epsilon_2^h \epsilon_3^h)^{\frac{1}{2}}$. The d-D mixing with $\epsilon_1^h \sim 0.03$ and $\epsilon_3^h \sim 0.03$ may saturate the bound for the B_d-$\bar{B}_d$ mixing via the Z-boson coupling, $r_1(B_d) \equiv |C_{B_d}^1|/|C_{B_d}^1|_{\rm max} \approx 1$, as seen in Fig. 5.3, which also provides significant effects $\sim 0.1\%$ on the flavor-diagonal Z-boson couplings in Eq. (5.48). On the other hand, as long as $\epsilon_i^h \lesssim 0.03$, the d-D mixing effect $C_{B_s}^1$ via the Z-boson coupling is fairly below the bound for the B_s-$\bar{B}_s$ mixing, $r_1(B_s) \equiv |C_{B_s}^1|/|C_{B_s}^1|_{\rm max} < 0.1$, as seen in Fig. 5.4. It should be noted here that as seen in Figs. 5.3 and 5.4, the contributions $C_{B_d}^2$ ($\circ$) and $C_{B_s}^2$ ($\blacktriangle$) via the singlet S_- coupling $\Lambda_{\mathcal{D}}^{S_-}[d^c d]$ of Eq. (5.34) may dominate over the Z-boson coupling effects in the parameter region of $\epsilon_i^f > \epsilon_i^h$, where the bounds in Eq. (5.43) are applicable. In particular, the case of $\epsilon_i^f \gg \epsilon_i^h$ is connected gradually to a suitable parameter region in the basis (a).

In short, remarkable effects may be provided particularly for the B mesons through the significant mixing between the b quark and the singlet D quark with $\epsilon_3^f \sim 1$ or $\epsilon_3^h \sim 0.03$, as seen in the above. They are fairly expected to serve as new physics for the flavor-changing processes and CP-violation in the B meson physics [5, 6, 7, 8, 10, 11, 13, 14, 16, 24, 25, 38, 60, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 85, 86]. Specifically, it is well known that there is tension between the experimental constraints and the standard model contribution to the $b \to s\gamma$ process, and hence this process should not be used as a constraint at present. The recent constraint by HFAG [87] is given as the average of the data of BABAR, Belle, and CLEO, $\mathrm{Br}(b \to s\gamma) = (352 \pm 23 \pm 9) \times 10^{-6}$, while the recent predictions of the standard model contribution are given as $\mathrm{Br}(b \to s\gamma) = (315 \pm 23) \times 10^{-6}$ [88] and $\mathrm{Br}(b \to s\gamma) = (298 \pm 26) \times 10^{-6}$ [89]. Even though this discrepancy is small, it may be confirmed by future experiments. The FCNC's via the d-D mixing can provide a

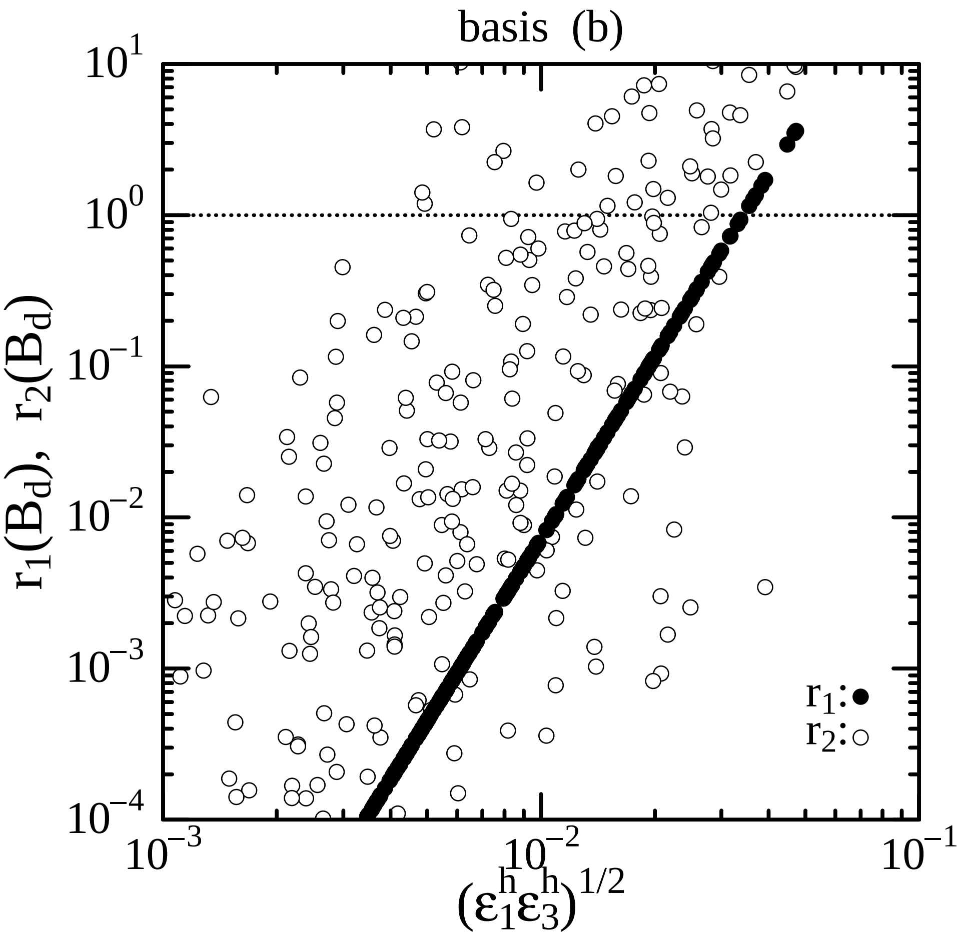

Figure 5.3: Scatter plots of $r_1(B_d) \equiv |C^1_{B_d}|/|C^1_{B_d}|_{\max}$ ($\bullet$) and $r_2(B_d) \equiv |C^2_{B_d}|/|C^2_{B_d}|_{\max}$ ($\circ$) versus $(\epsilon^h_1\epsilon^h_3)^{1/2}$ for the B_d-$\bar{B}_d$ mixing, which are provided by the Z coupling $\Delta\mathcal{Z}_{\mathcal{D}}[d^\dagger d]$ and the S_- coupling $\Lambda^{S_-}_{\mathcal{D}}[d^c d]$, respectively, in the basis (b).

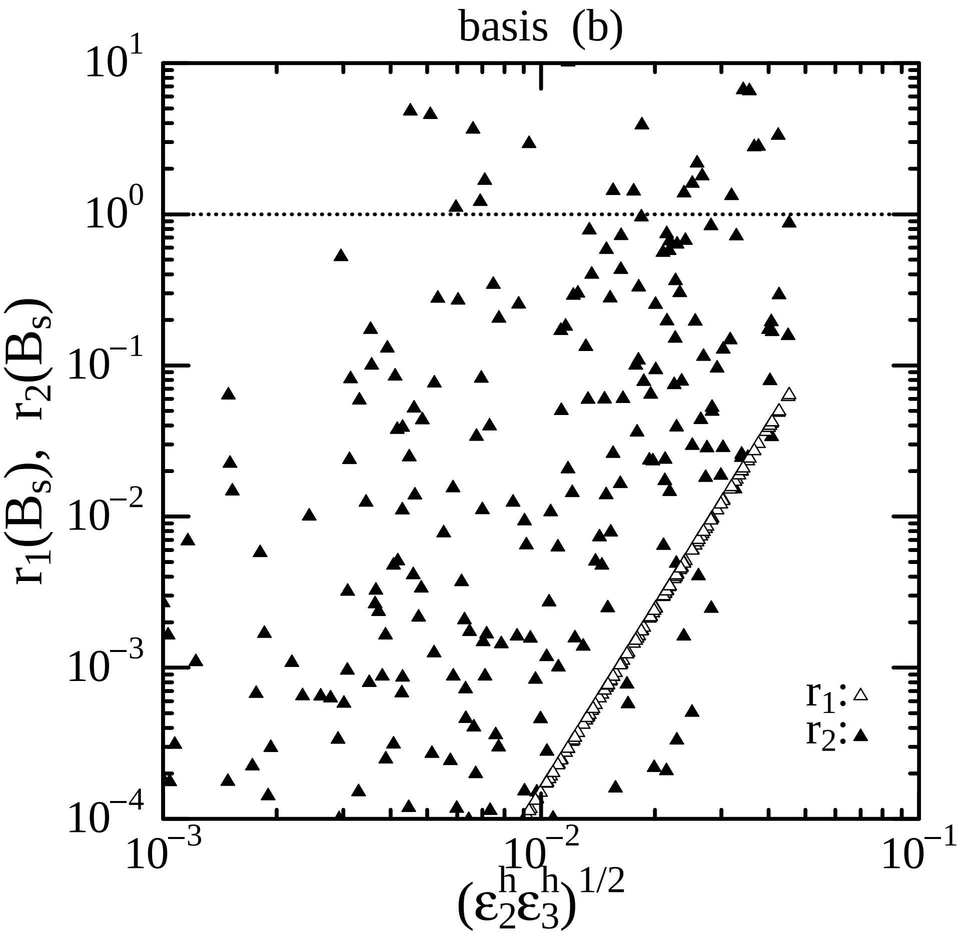

Figure 5.4: Scatter plots of $r_1(B_s) \equiv |C^1_{B_s}|/|C^1_{B_s}|_{\max}$ (△) and $r_2(B_s) \equiv |C^2_{B_s}|/|C^2_{B_s}|_{\max}$ (▲) versus $(\epsilon_2^h\epsilon_3^h)^{1/2}$ for the B_s-$\bar{B}_s$ mixing, similarly to Fig. 5.3.

solution of the discrepancy.

5.4 Decays of singlet quarks and Higgs particles

We now investigate the decays of the singlet quarks and Higgs particles, which will provide distinct signatures upon their productions at the LHC. The flavor-changing interactions between the singlet quarks Q and the ordinary quarks q are relevant for these decays at the tree level. Specifically, the left-handed Z-boson couplings are given as

$$\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q]_{ia} = \mathcal{Z}_{\mathcal{Q}}[Q^\dagger q]^*_{ai} = (V^\dagger_{q_{\rm L}} \epsilon_{q_{\rm L}})_{ia} \simeq (\epsilon_{q_{\rm L}})_{ia}, \tag{5.51}$$

while the right-handed ones are absent. Note here that $\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q] = \Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q]$, as shown in Eq. (5.90); the q-Q transitions in the Z-boson couplings are just induced as the q-Q mixing effect. The left-handed W-boson couplings are given in terms of the Z-boson couplings and the CKM matrix in a good approximation up to the second order of the small q-Q mixing as

$$\begin{aligned} \mathcal{V}[u^\dagger D]_{ia} &= \mathcal{V}[D^\dagger u]^*_{ai} \\ &\simeq (V \mathcal{Z}_{\mathcal{D}}[d^\dagger D])_{ia}, \end{aligned} \tag{5.52}$$

$$\begin{aligned} \mathcal{V}[d^\dagger U]_{ia} &= \mathcal{V}[U^\dagger d]^*_{ai} \\ &\simeq (V^\dagger \mathcal{Z}_{\mathcal{U}}[u^\dagger U])_{ia}, \end{aligned} \tag{5.53}$$

while the right-handed ones are absent. The neutral scalar couplings are given as

$$\Lambda^H_{\mathcal{Q}}[q^c Q]_{ia} = (m_{q_i}/v)\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q]_{ia}, \tag{5.54}$$

$$\Lambda^H_{\mathcal{Q}}[Q^c q]_{ai} = (m_{Q_a}/v)\mathcal{Z}_{\mathcal{Q}}[Q^\dagger q]_{ai}, \tag{5.55}$$

$$\Lambda^{S_+}_{\mathcal{Q}}[q^c Q]_{ia} = -(m_{q_i}/v_S)\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q]_{ia}, \tag{5.56}$$

$$\Lambda^{S_+}_{\mathcal{Q}}[Q^c q]_{ai} = -(m_{Q_a}/v_S)\mathcal{Z}_{\mathcal{Q}}[Q^\dagger q]_{ai}, \tag{5.57}$$

$$\Lambda^{S_-}_{\mathcal{Q}}[q^c Q]_{ia} = (V^\dagger_{q_{\rm R}} f^-_Q V_{Q_{\rm L}} - \epsilon'_{q_{\rm R}} \lambda^-_Q V_{Q_{\rm L}})_{ia}/\sqrt{2}, \tag{5.58}$$

$$\Lambda^{S_-}_{\mathcal{Q}}[Q^c q]_{ai} = (-\epsilon^\dagger_{q_{\rm R}} f^-_Q \epsilon'^\dagger_{q_{\rm L}} - V^\dagger_{Q_{\rm R}} \lambda^-_Q \epsilon'^\dagger_{q_{\rm L}})_{ai}/\sqrt{2}. \tag{5.59}$$

Here, $\Lambda^{\phi^0_r}_{\mathcal{Q}}[q^c Q]_{ia}$ stands for $\bar{q}_{i{\rm R}} Q_{a{\rm L}} \phi^0_r$, and $\Lambda^{\phi^0_r}_{\mathcal{Q}}[Q^c q]_{ai}$ for $\bar{Q}_{a{\rm R}} q_{i{\rm L}} \phi^0_r$, respectively, in terms of the Dirac fields. The relations among the gauge and scalar couplings in Eqs. (5.51) – (5.57) are derived in the Appendix B.

5.4.1 Singlet quark decays

We first investigate the singlet quark decays. We describe the essential features by considering the case of one down-type singlet quark D ($a = 1$ is omitted for $N_D = 1$ and $N_U = 0$). Similar results are obtained in the general cases of some D and U quarks, especially for the lightest singlet quark. While the heavier singlet quarks may decay dominantly into the lighter singlet quarks and Higgs particles in the general cases, we concentrate on the decays of the lightest singlet quark producing the ordinary quarks.

The partial widths of the relevant decay modes are calculated (when they are kinematically allowed) as

$$
\begin{aligned}
\Gamma(D \to u_i W) &= \frac{G_F}{\sqrt{2}} \frac{m_D^3}{8\pi} g(x_W, x_{u_i}) |\mathcal{V}[u^\dagger D]_i|^2, && (5.60)\\
\Gamma(D \to d_i Z) &= \frac{G_F}{\sqrt{2}} \frac{m_D^3}{16\pi} (1 - 3x_Z^4 + 2x_Z^6) |\mathcal{Z}_\mathcal{D}[d^\dagger D]_i|^2, && (5.61)\\
\Gamma(D \to d_i H) &= \frac{m_D}{16\pi} (1 - x_H^2)^2 (|\Lambda_\mathcal{D}^H [D^c d]_i|^2 + |\Lambda_\mathcal{D}^H [d^c D]_i|^2), && (5.62)\\
\Gamma(D \to d_i S_\pm) &= \frac{m_D}{16\pi} (1 - x_{S_\pm}^2)^2 (|\Lambda_\mathcal{D}^{S_\pm} [D^c d]_i|^2 + |\Lambda_\mathcal{D}^{S_\pm} [d^c D]_i|^2), && (5.63)
\end{aligned}
$$

where $x_W = m_W/m_D$, $x_Z = m_Z/m_D$, $x_H = m_H/m_D$, $x_{S_\pm} = m_{S_\pm}/m_D$, $x_{u_i} = m_{u_i}/m_D$, and

$$
g(x, y) = [1 - (x + y)^2]^{1/2} [1 - (x - y)^2]^{1/2} [x^2(1 - 2x^2 + y^2) + (1 - y^2)^2]. \quad (5.64)
$$

The scalar mixing is neglected ($O_\phi = \mathbf{1}$) for a while. The kinematic effects of $m_{d_i}/m_D \lesssim 0.02$ for $m_D \gtrsim 200\text{GeV}$ may be neglected in a good approximation, while $\Gamma(D \to tW)$ depends sensibly on m_t/m_D.

The flavor-structure of the d-D mixing is measured manifestly in the D decays into the ordinary quarks $d_i = d, s, b$ and the Z boson as

$$
\Gamma(D \to d_i Z) \propto |(\epsilon_{d_\mathrm{L}})_i|^2, \quad (5.65)
$$

where $\mathcal{Z}_\mathcal{D}[d^\dagger D]_i \simeq (\epsilon_{d_\mathrm{L}})_i$ for the Z-boson coupling. The partial width of the D quark decays producing the Z boson is inclusively estimated for the reference as

$$
\begin{aligned}
\Gamma_D(Z) &\equiv \sum_i \Gamma(D \to d_i Z) \\
&\sim 20\text{MeV} \times \left(\frac{m_D}{500\text{GeV}} \right)^3 \frac{|\epsilon^h|^2}{(0.03)^2}, && (5.66)
\end{aligned}
$$

where $(\epsilon_{d_{\rm L}})_i \sim \epsilon^h_i$ with $|\epsilon^h| \equiv [(\epsilon^h_1)^2+(\epsilon^h_2)^2+(\epsilon^h_3)^2]^{1/2}$ in the basis (b), and $\epsilon^h_i \rightarrow (m_{d_i}/m_D)\epsilon^f_i$ for $(\epsilon_{d_{\rm L}})_i$ in the basis (a). By considering Eqs. (5.51), (5.52), (5.54) and (5.55) with $|\Lambda^H_{\mathcal{D}}[d^c D]_i|^2/|\Lambda^H_{\mathcal{D}}[D^c d]_i|^2 = (m_{d_i}/m_D)^2 \ll 1$, we find the well-known relations [77, 78]

$$\Gamma(D \rightarrow u_i W) \sim 2\Gamma(D \rightarrow d_i Z), \tag{5.67}$$

$$\Gamma(D \rightarrow d_i H) \sim \Gamma(D \rightarrow d_i Z), \tag{5.68}$$

or inclusively

$$\Gamma_D(W) \equiv \sum_i \Gamma(D \rightarrow u_i W) \sim 2\Gamma_D(Z), \tag{5.69}$$

$$\Gamma_D(H) \equiv \sum_i \Gamma(D \rightarrow d_i H) \sim \Gamma_D(Z), \tag{5.70}$$

where $O_\phi = \mathbf{1}$. The actual values of these widths are evaluated depending on the kinematic factors and the CKM mixing.

In the present model with the complex singlet Higgs, the decays of the singlet quark D producing the singlet Higgs scalars $S_\pm$ are possible for $m_D > m_{S_\pm} + m_{d_i}$. Especially, it is remarkable that the decays with S_- may dominate over the other decay modes. By considering $(f^-_D)_i \sim (\epsilon'_{d_{\rm R}})_i \lambda^-_D \sim (m_D/v_S)\epsilon^f_i$ and $\lambda^-_D(\epsilon'^\dagger_{d_{\rm L}})_i \sim (m_D/v_S)\epsilon^h_i$ in Eqs. (5.58) and (5.59), the S_- couplings are estimated roughly as

$$\Lambda^{S_-}_{\mathcal{D}}[d^c D]_i \sim (m_D/v_S)\epsilon^f_i, \tag{5.71}$$

$$\Lambda^{S_-}_{\mathcal{D}}[D^c d]_i \sim (m_D/v_S)\epsilon^h_i. \tag{5.72}$$

Here, for the sake of convenience we adopt $\epsilon^h_i = (m_{d_i}/m_D)\epsilon^f_i$ in the basis (a) though $h_d = \mathbf{0}$, while $\epsilon^f_i = (v_S/m_D)|2(f_Q)_i|/\sqrt{2}$ in the basis (b) though $f^+_Q = \mathbf{0}$, as discussed concerning Eq. (5.35). Then, we estimate roughly the partial width of the D decays producing S^- as

$$\begin{aligned}\Gamma_D(S_-) &\equiv \sum_i \Gamma(D \rightarrow d_i S_-) \\ &\sim 10\text{MeV} \times \left(\frac{m_D}{500\text{GeV}}\right)^3 \left(\frac{500\text{GeV}}{v_S}\right)^2 \frac{|\epsilon^f|^2 + |\epsilon^h|^2}{(0.03)^2},\end{aligned} \tag{5.73}$$

where $|\epsilon^f| \equiv [(\epsilon^f_1)^2 + (\epsilon^f_2)^2 + (\epsilon^f_3)^2]^{1/2}$. (The actual value is reduced to some extent by the kinematic factor for $m_D \sim m_{S^-}$.) This width $\Gamma_D(S_-)$ for the decays into the singlet scalar S_- dominates over the reference width $\Gamma_D(Z)$ for the decays into the Z boson in

Eq. (5.66) for $|\epsilon^f|^2 \gg |\epsilon^h|^2$, especially in the basis (a) with $\epsilon_i^h \to (m_{d_i}/m_D)\epsilon_i^f$. As for the D decays with S_+, the partial width is simply related to $\Gamma_D(Z)$ by Eq. (5.57) as

$$\Gamma_D(S_+) \equiv \sum_i \Gamma(D \to d_i S_+) \sim (v/v_S)^2 \Gamma_D(Z), \tag{5.74}$$

which amounts to $O(10\%)$ of $\Gamma_D(Z)$ for $v_S \approx 500\text{GeV}$. Even this slight enhancement due to $\Gamma_D(S_+)$ for the D decays into the scalars H and S_+ might serve as an experimental signature for the singlet Higgs even if S_- is absent in the model with one real $S \equiv S_+$.

Here, it should be noted that the S_- coupling may even provide significant contributions to the decays $D \to d_i H$ through the H-S_- mixing ϵ_{HS_-} in O_ϕ. In fact, the d-D coupling with the standard Higgs H (more precisely the mass eigenstate $\phi_1 \simeq H$ with $\epsilon_{HS_-} \ll 1$) is replaced in Eq. (5.62) as

$$\Lambda_{\mathcal{D}}^H \to \Lambda_{\mathcal{D}}^H + \epsilon_{HS_-} \Lambda_{\mathcal{D}}^{S_-}. \tag{5.75}$$

Then, instead of Eq. (5.70) we obtain

$$\Gamma_D(H) \;\; \sim \;\; \Gamma_D(Z) + \epsilon_{HS_-}^2 \Gamma_D(S_-), \tag{5.76}$$

where the interference term between $\Lambda_{\mathcal{D}}^H$ and $\Lambda_{\mathcal{D}}^{S_-}$ is omitted for simplicity. This enhancement of $\Gamma_D(H)$ with $\epsilon_{HS_-}\Lambda_{\mathcal{D}}^{S_-}$ in Eq. (5.75) is valid even when the decays $D \to d_i S_-$ are forbidden kinematically for $m_D < m_{S_-} + m_{d_i}$. In the presence of a small but sizable H-S_- mixing $\epsilon_{HS_-} \sim 0.01 - 0.1$ the singlet quark decays $D \to d_i H$ may become the dominant modes, particularly in the basis (a) due to the substantial suppression of $D \to d_i Z$ with $|\mathcal{Z}_{\mathcal{D}}[d^\dagger D]_i|^2 \sim (m_{d_i}/m_D)^2 (\epsilon_i^f)^2 \lesssim 10^{-4}$. Hence, if $\Gamma_D(H) \gg \Gamma_D(Z)$ is confirmed experimentally, which is contrary to the usual expectation of Eq. (5.70), it will provide a distinct evidence for the complex singlet Higgs field S with the H-S_- mixing. The decays $D \to d_i S_+$ may also be enhanced substantially as $\Gamma_D(S_+) \sim \epsilon_{S_+S_-}^2 \Gamma_D(S_-)$ via a sizable S_+-S_- mixing $\epsilon_{S_+S_-}$.

We present in the following the detailed estimates on the widths of the relevant decay modes, where the constraints on the d-D mixing from the $\Delta F = 2$ meson mixings and the diagonal Z-boson couplings are checked to be satisfied according to the numerical calculations performed in Sec. 5.3.

We suitably denote the ratios of the relevant widths to the reference width as

$$R_D(X/Z) \equiv \frac{\Gamma_D(X)}{\Gamma_D(Z)}, \tag{5.77}$$

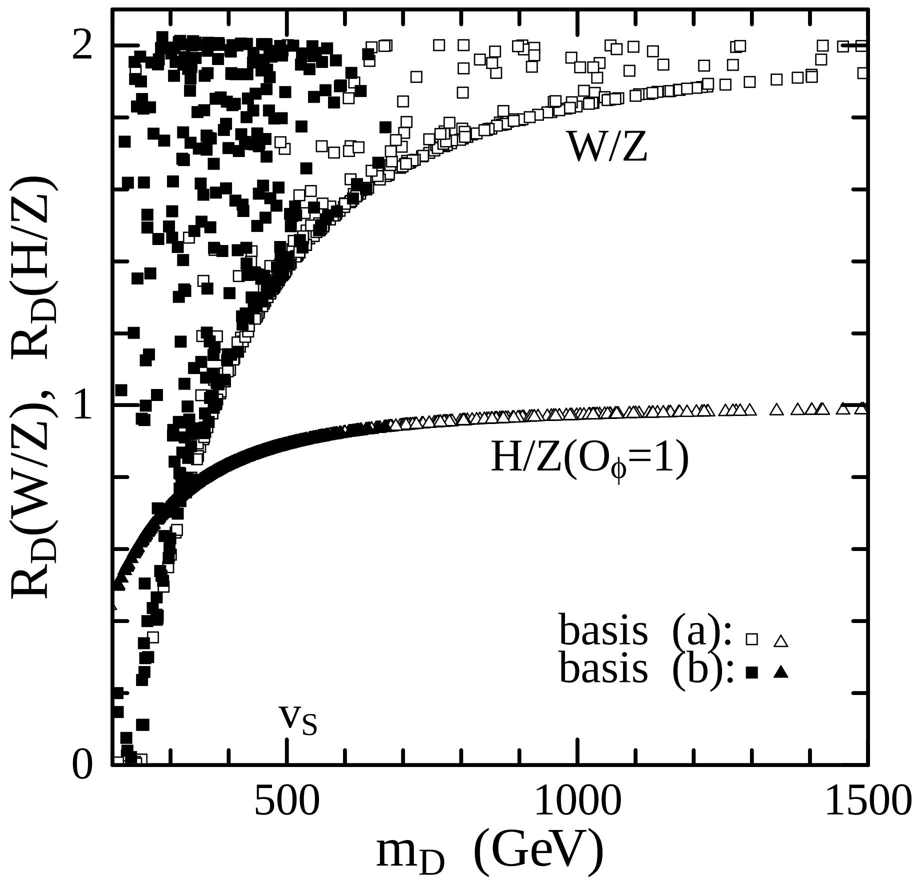

Figure 5.5: $R_D(W/Z) \equiv \Gamma_D(W)/\Gamma_D(Z)$ versus m_D is shown for the bases (a) (□) and (b) (■). $R_D(H/Z) \equiv \Gamma_D(H)/\Gamma_D(Z)$ versus m_D is also shown for the bases (a) (△) and (b) (▲). Here, $v_S = 500$GeV, and the Higgs scalar mixing is assumed to be absent ($O_\phi = \mathbf{1}$).

where $X = W, H, S_+, S_-$. For the usual decay modes $D \to u_i W$, $D \to d_i Z$ and $D \to d_i H$, scatter plots of $R_D(W/Z)$ (□, ■) and $R_D(H/Z)$ (△, ▲) versus the singlet quark mass m_D are shown in Fig. 5.5 for the bases (a) (□, △) and (b) (■, ▲). Here, $v_S = 500\text{GeV}$, and the Higgs scalar mixing is assumed to be absent ($O_\phi = \mathbf{1}$). Similar results are obtained for the bases (a) and (b) since these bases are equivalently related to each other by the unitary transformation, as discussed in Sec. 5.2. Note here that larger values may be obtained for the singlet quark mass m_D with a given singlet Higgs VEV v_S in the basis (a) (□, △), which is due to the significant contribution of the q-Q mixing term $\Delta_{qQ} = f_Q^+ v_S/\sqrt{2}$ for $|\epsilon^f| \sim 1$. The lower boundary curve for $R_D(W/Z)$ reflects the kinematic factor of the dominant top contribution $D \to tW$ with $|\mathcal{V}[u^\dagger D]_3|^2 \gg |\mathcal{V}[u^\dagger D]_{1,2}|^2$. In this case the singlet D quark mixes mainly with the b quark as $|(\epsilon_{d_\mathrm{L}})_3|^2 \gg |(\epsilon_{d_\mathrm{L}})_{1,2}|^2$. On the other hand, in the case that the top contribution is negligible with $|\mathcal{V}[u^\dagger D]_3|^2 \ll |\mathcal{V}[u^\dagger D]_{1,2}|^2$, the asymptotic value $R_D(W/Z) = 2$ is almost saturated for $m_D \gtrsim 300\text{GeV}$. We also see that $R_D(H/Z)$ approaches the asymptotic value $R_D(H/Z) = 1$ showing the kinematic dependence on m_D. These results really confirm the usual expectation in Eqs. (5.69) and (5.70). It should, however, be remarked that as shown in Eq. (5.76), the D decays with the standard Higgs H may be enhanced substantially as $R_D(H/Z) \gg 1$ due to the singlet Higgs coupling $\Lambda_{\mathcal{D}}^{S_-}$ via the H-S_- mixing.

The reference width $\Gamma_D(Z)$ versus the magnitude of the left-handed d-D mixing $|\epsilon^h|$, as given in Eq. (5.66), is shown in Fig. 5.6. Here, the marks ○ and ● denote the estimates in the bases (a) and (b), respectively, and $\epsilon_i^h = (m_{q_i}/m_D)\epsilon_i^f$ as Eq. (5.35) is adopted in the basis (a) though $\epsilon_i^h = 0$ formally. This plot of $\Gamma_D(Z)$ spreads according to the variation of $m_D \sim 100\text{GeV} - 1\text{TeV}$ due to the fact that $\Gamma_D(Z)$ is almost proportional to m_D^3.

The decay widths $\Gamma_D(Z)$ and $\Gamma_D(S_-)$ for the significant modes are compared in Fig. 5.7. According to Eqs. (5.66) and (5.73), by measuring these decay widths we can estimate the magnitudes of d-D mixings, $|\epsilon^h|$ from the h_d coupling and $|\epsilon^f|$ from the f_D and f_D' couplings. Specifically, $\Gamma_D(S_-) \gg \Gamma_D(Z)$ for $|\epsilon^f| \gg |\epsilon^h|$ as in the basis (a) (○), while $\Gamma_D(S_-) \lesssim \Gamma_D(Z)$ for $|\epsilon^f| \lesssim |\epsilon^h|$ as in the basis (b) (●). The decay width $\Gamma_D(H)$ with the standard Higgs H is also relevant to measure the relative significance of $|\epsilon^h|$ versus $|\epsilon^f|$ according to Eq. (5.76) with the sizable H-S_- mixing. This is useful even if the decays $D \to d_i S_-$ are kinematically forbidden for $m_{S_-} > m_D + m_{d_i}$. A plot of $R_D(H/Z)$ versus $|\epsilon^f|/|\epsilon^h|$ is shown in Fig. 5.8 for the bases (a) (△) and (b) (▲), where $\epsilon_{HS_-} = 0.1$ is taken typically for the H-S_- mixing. In the region of $|\epsilon^f|/|\epsilon^h| \gg 1$, the contribution of the

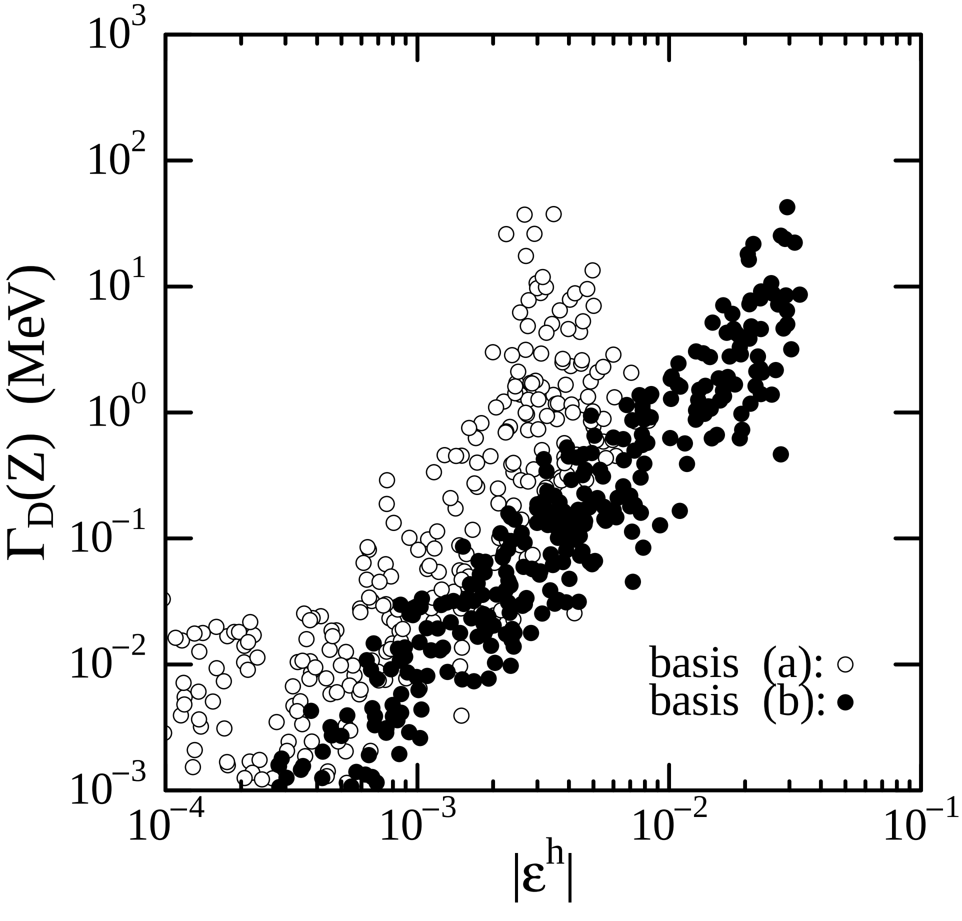

Figure 5.6: $\Gamma_D(Z)$ versus $|\epsilon^h|$ in the bases (a) ($\circ$) and (b) ($\bullet$). Here, $\epsilon_i^h = (m_{q_i}/m_D)\epsilon_i^f$ as Eq. (5.35) is adopted in the basis (a) though $\epsilon_i^h = 0$ formally.

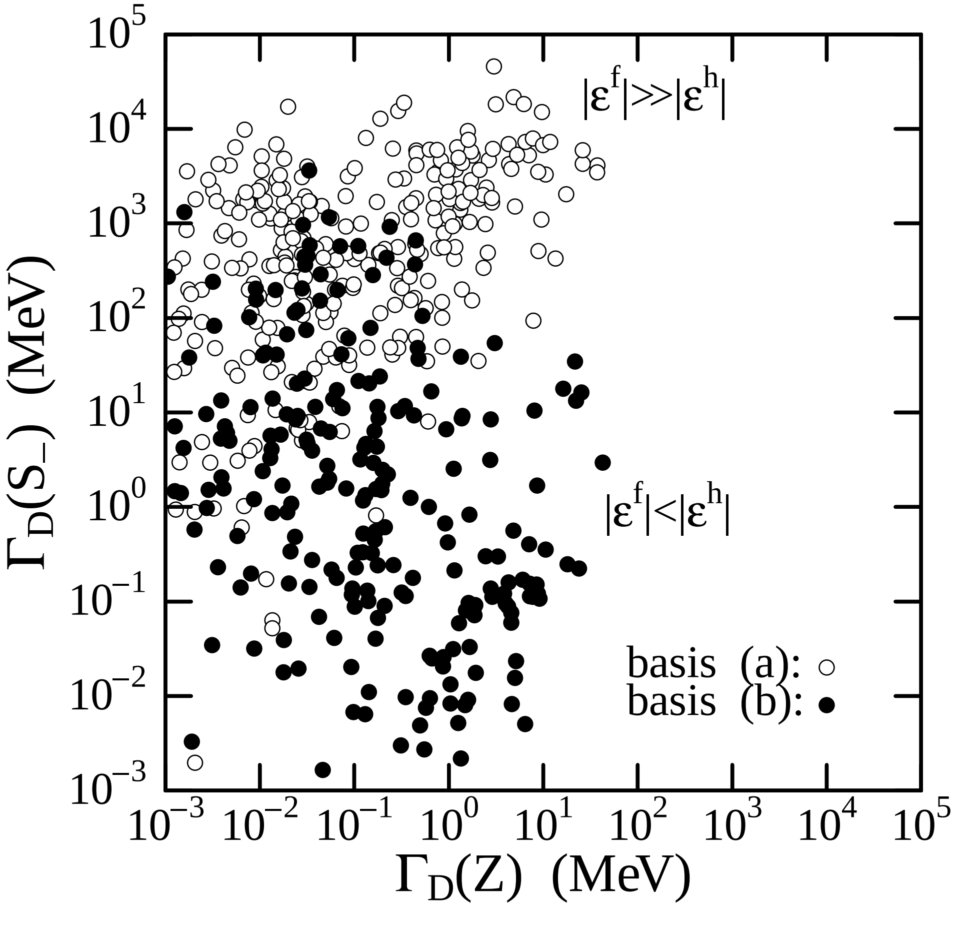

Figure 5.7: $\Gamma_D(Z)$ and $\Gamma_D(S_-)$ are compared in the bases (a) ($\circ$) and (b) ($\bullet$).

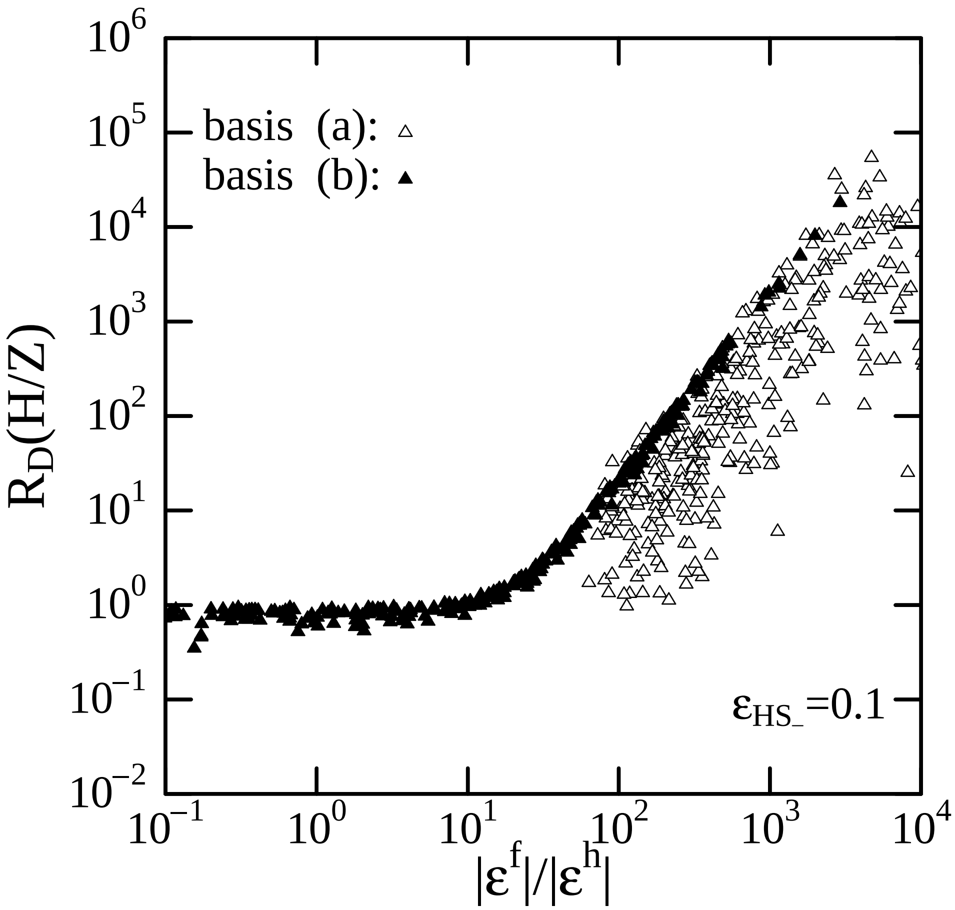

Figure 5.8: $R_D(H/Z) \equiv \Gamma_D(H)/\Gamma_D(Z)$ versus $|\epsilon^f|/|\epsilon^h|$ is shown for the bases (a) ($\triangle$) and (b) ($\blacktriangle$), where $\epsilon_{HS_-} = 0.1$ is taken typically for the H-S_- mixing.

singlet Higgs coupling $\Lambda_{\mathcal{D}}^{S_-}$ dominates as $\Gamma_D(H) \sim \epsilon_{HS^-}^2 \Gamma_D(S_-) \gg \Gamma_D(Z)$. On the other hand, in the region of $|\epsilon^f|/|\epsilon^h| \lesssim 1$ we have $\Gamma_D(H) \sim \Gamma_D(Z)$ as usually expected.

In these plots of Figs. 5.5, 5.6, 5.7 and 5.8, the regions for the bases (a) and (b) overlap as expected, but they are not identical. This is because the actual parameter ranges are somewhat different for the bases (a) and (b); although the parameter ranges have been taken apparently in the same way for these bases in the numerical calculations, except that $h_q = \mathbf{0}$ in the basis (a), they are not mapped identically to each other by the unitary transformation between the bases (a) and (b). Specifically, in the basis (a) we have a significant constraint $|\epsilon^h|/|\epsilon^f| \lesssim m_b/m_D \sim 0.01$ from the relation $\epsilon_i^h \sim (m_{d_i}/m_D)\epsilon_i^f$, implying $|\epsilon^h| \lesssim 0.01$ as long as $|\epsilon^f| \lesssim 1$. This is explicitly seen in Figs. 5.6 and 5.8. We also note that the plot for the basis (a) in Fig. 5.8 spreads significantly. This is in some sense an artifact due to the definition of the d-D mixing parameters ϵ_i^f in terms of $f_D^+ \equiv f_D + f_D'$ for the basis (a). The singlet S_- coupling $\Lambda_{\mathcal{D}}^{S_-}$ is rather given by $f_D^- \equiv i(f_D - f_D')$. The spread in the plot for the basis (a) really reflects the partial cancellation between f_D and f_D' for the $\Lambda_{\mathcal{D}}^{S_-}$ coupling. On the other hand, for the basis (b) the parameters ϵ_i^f are defined suitably with $f_D^- = 2if_D$ ($f_D = -f_D'$ for $f_D^+ = \mathbf{0}$). Hence, the plot for the basis (b) almost lies on a curve up to the small fluctuation due to the kinematic factor, which gives the boundary of the plot for the basis (a). This boundary really corresponds to the extreme case $f_D \approx -f_D'$ for $|f_D^-| \approx 2|f_D|$ in the basis (a).

As seen so far, the singlet D quark decays present us important insights on the d-D mixing effects for the flavor-changing processes. Especially, if it is observed that $\Gamma_D(S_-) \gg \Gamma_D(Z)$, we find that the singlet Higgs scalar interactions dominate over the Z-boson interactions. For example, suppose that the current experimental bound for the B_d-$\bar{B}_d$ mixing [84] is almost saturated with $(\epsilon_1^f \epsilon_3^f)^{\frac{1}{2}} \sim 0.2$ for $m_{S_-} \sim 300\text{GeV}$ in the basis (a) as shown in Eq. (5.42), which implies $|\epsilon^f| \gtrsim \sqrt{2} \times 0.2 \gg |\epsilon^h|$. Then, we expect $\Gamma_D(S_-) \sim 1\text{GeV} - 10\text{GeV} \gg \Gamma_D(Z)$ for $m_D \sim 500\text{GeV} - 1\text{TeV}$, as seen in Eq. (5.73) and Fig. 5.7. Contrarily, if $\Gamma_D(S_-) \lesssim 1\text{MeV}$ for $m_D \gtrsim 500\text{GeV}$, which implies $|\epsilon^f| \lesssim 0.01$, the scalar FCNC's do not provide significant contributions to the $\Delta F = 2$ meson mixings. As for the D decays with the Z boson, if there is a significant left-handed d-D mixing as $|\epsilon^h| \sim 0.03$ in the basis (b), we expect $\Gamma_D(Z) \sim 10\text{MeV} - 100\text{MeV}$ for $m_D \sim 500\text{GeV} - 1\text{TeV}$ in Eq. (5.66). In this case, the bound for the B_d-$\bar{B}_d$ mixing may be saturated by the Z-mediated FCNC with $(\epsilon_1^h \epsilon_3^h)^{\frac{1}{2}} \sim 0.03$, as shown in Eq. (5.44). On the other hand, if $\Gamma_D(Z) \lesssim 0.01\text{MeV}$ for $m_D \gtrsim 500\text{GeV}$, which implies $|\epsilon^h| \lesssim 0.001$ (see Fig. 5.6), the effects of the Z-mediated FCNC's are negligible in the $\Delta F = 2$ meson mixings.

5.4.2 Higgs particle decays

We next survey the decays of the Higgs particles H, S_+ and S_-, or more precisely the mass eigenstates ϕ_1, ϕ_2, ϕ_3 with the mixing matrix O_ϕ.

The standard Higgs H is probably lighter than the singlet quarks Q ($m_Q > m_H \approx$ 120GeV) so that its decays involving the singlet quarks are forbidden kinematically. It should also be noted that the q-Q mixing effect on the H coupling with the ordinary quarks appears merely at the second order related to the modification of the Z-boson coupling, as seen in Eq. (5.31). Hence, the Higgs particle H will decay essentially in the same way as the standard model unless the H-S_- mixing is so large as to provide significant effects.

The singlet Higgs particles $S_\pm$ will be produced significantly by gluon fusion via a loop of singlet quark Q coupled to $S_\pm$ with the strength $\sim |\lambda_Q^\pm| \sim m_Q/v_S \sim 1$. The production rates of $S_\pm$ will be comparable to that of the standard Higgs H unless $S_\pm$ are substantially heavier than H. If $m_{S_\pm} < m_Q$, the singlet quark decays $Q \to q_i S_\pm$ also produce $S_\pm$, as discussed so far. It should be remarked that some indirect indication for the presence of S_- may be obtained via the H-S_- mixing, specifically in the case of $\Gamma_Q(H) \gg \Gamma_Q(Z)$ for the singlet quark decays $Q \to qH$.

In the case of $m_{S_\pm} < m_Q$, the singlet Higgs particles $S_\pm$ decay predominantly into the ordinary quarks through the scalar interactions in Eqs. (5.32), (5.33) and (5.34) at the second order of the q-Q mixing:

$$S_\pm \to q_i \bar{q}_j. \tag{5.78}$$

The decay widths are estimated particularly for S_- in comparison with that of the standard Higgs H as

$$\begin{aligned}\frac{\Gamma(S_- \to q_i\bar{q}_j)}{\Gamma(H \to b\bar{b})} &\sim \frac{m_{S_-}(|\Lambda_{\mathcal{Q}}^{S_-}[q^c q]_{ij}|^2 + |\Lambda_{\mathcal{Q}}^{S_-}[q^c q]_{ji}|^2)}{m_H|\Lambda_{\mathcal{Q}}^{H}[q^c q]_{33}|^2} \\ &\sim [(m_{S_-}/m_H)(m_Q/v_S)^2/(m_b/v)^2] \\ &\quad \times[(\epsilon_i^f)^2(\epsilon_j^h)^2 + (\epsilon_j^f)^2(\epsilon_i^h)^2],\end{aligned} \tag{5.79}$$

where $\epsilon_j^h \to (m_{q_j}/m_Q)\epsilon_j^f$ in the basis (a). We estimate, for instance, $\Gamma(S_- \to b\bar{b})/\Gamma(H \to b\bar{b}) \sim [10(\epsilon_3^f\epsilon_3^h)^{\frac{1}{2}}]^4$ for $m_{S_-}/m_H \sim 3$, $m_D/v_S \sim 1$ and $m_b/v \simeq 1/60$, which may amount to

$O(1)$ for the large b-D mixing as $\epsilon_3^f \sim 1$ and $\epsilon_3^h \sim (m_b/m_D)\epsilon_3^f \sim 0.01$. The flavor-changing decays such as $S_- \to b\bar{s}$ as well as the flavor-diagonal ones may have significant fractions. This is distinct from the standard Higgs H, presenting a promising signature of the singlet Higgs S_-. In fact, we estimate

$$\frac{\Gamma(S_- \to b\bar{d}_j)}{\Gamma(S_- \to b\bar{b})} \sim (\epsilon_j^f/\epsilon_3^f)^2 + (\epsilon_j^h/\epsilon_3^h)^2, \tag{5.80}$$

depending on the flavor structure of the d-D mixing. If the singlet U quarks are present with a large t-U mixing, the decays $S_- \to t\bar{t}, t\bar{u}_i, u_i\bar{t}$ involving the top quark may be observed with significant fractions.

In this way, the decays of the singlet Higgs S_- into the ordinary quarks are determined in terms of the q-Q mixing parameters with close connection to the flavor-changing processes such as the $\Delta F = 2$ meson mixings. As for the the singlet Higgs S_+, its coupling is given in Eq. (5.32) by the Z-boson coupling at the second order of q-Q mixing with further suppression by the ordinary quark mass. These arguments on the $S_\pm$ couplings with the ordinary quarks generally suggest that

$$\Gamma_H \gtrsim \Gamma_{S-} \gg \Gamma_{S_+}(m_{S_\pm} < m_Q, O_\phi \approx \mathbf{1}). \tag{5.81}$$

for the decay rates of the Higgs particles if the Higgs mixing is negligibly small. It is, however, possible that the large Higgs mixing, in cooperation with the q-Q mixing, affects significantly the decays of H and $S_\pm$. (See also Ref. [90, 91, 92] for investigations of extended Higgs models at the LHC.) Therefore, the observations of the Higgs particle decays present important information on the Higgs mixing and q-Q mixing.

In the case of $m_{S_\pm} > m_Q$, the singlet quark decays $Q \to qS_\pm$ are forbidden kinematically. Even in such a case the singlet Higgs $S_\pm$ will be produced significantly by the gluon fusion via the singlet quark loop. Then, they decay predominantly involving the singlet quarks as

$$S_\pm \to Q\bar{q}, \bar{Q}q, Q\bar{Q}. \tag{5.82}$$

The decay widths are estimated in terms of the scalar couplings $\Lambda_Q^{S_\pm}$ in Eq. (5.27). In particular, if $m_{S_\pm} > 2m_Q$ we have

$$\Gamma(S_\pm \to Q\bar{Q}) \sim \frac{(m_Q/v_S)^2 m_{S_\pm}}{16\pi} \gtrsim 10\text{GeV} \gg \Gamma_H \tag{5.83}$$

with $\mathrm{Br}(S_\pm \to Q\bar{Q}) \approx 1$ for $m_{S_\pm} \gtrsim 500$GeV and $|\lambda_Q^\pm| \sim m_Q/v_S \sim 1$.

5.5 Summary

The singlet quarks in cooperation with the single Higgs field may provide various interesting effects in particle physics and cosmology through the mixing with the ordinary quarks (q-Q mixing). It is hence worth considering their phenomenological implications toward the discovery of them at the LHC. In this chapter we have investigated the flavor-changing interactions in the model with singlet quarks and singlet Higgs, which are induced by the q-Q mixing. While the gauge interactions have been investigated extensively in the literature, we have rather noted here that the scalar interactions mediated by the singlet Higg may provide significant effects in some cases. This possibility has not been paid so much attention before in the models with singlet quarks. We have considered the effects of the gauge and scalar interactions in the $\Delta F = 2$ mixings of the neutral mesons to show the currently allowed range of the q-Q mixing. Then, we have investigated the decays of the singlet quarks and Higgs particles as the new physics around the electroweak scale to the TeV scale, which is accessible to the LHC. Especially, the right-handed q-Q mixing may be tolerably large without contradicting the current bounds on the flavor-changing processes, since it is not involved directly in the electroweak gauge interactions. If this is the case, the scalar coupling by the singlet Higgs, and possibly through the Higgs mixing, provides distinct signatures for the decays of the singlet quarks and Higgs particles, which should be compared with the conventionally expected ones via the gauge and standard Higgs couplings. We expect that observations of the singlet quarks and Higgs particles will present us important insights on the q-Q mixing and Higgs mixing.

Appendix B
Relations among the gauge and scalar couplings

We here derive the suitable relations among the gauge and scalar couplings.

The full mixing matrix for the left-handed W-boson coupling is given by

$$\begin{aligned} \mathcal{V} &= \mathcal{V}_{\mathcal{U}_{\mathrm{L}}}^\dagger \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{D}_{\mathrm{L}}} \\ &= \begin{pmatrix} V_{u_{\mathrm{L}}}^\dagger V_0 V_{d_{\mathrm{L}}} & V_{u_{\mathrm{L}}}^\dagger V_0 \epsilon_{d_{\mathrm{L}}} \\ \epsilon_{u_{\mathrm{L}}}^\dagger V_0 V_{d_{\mathrm{L}}} & \epsilon_{u_{\mathrm{L}}}^\dagger V_0 \epsilon_{d_{\mathrm{L}}} \end{pmatrix} . \end{aligned} \tag{5.84}$$

Then, we obtain from the off-diagonal blocks

$$
\begin{aligned}
\mathcal{V}[u^\dagger D] &= V_{u_{\rm L}}^\dagger V_0 \epsilon_{d_{\rm L}} \simeq V V_{d_{\rm L}}^\dagger \epsilon_{d_{\rm L}}, && (5.85)\\
\mathcal{V}[U^\dagger d] &= \epsilon_{u_{\rm L}}^\dagger V_0 V_{d_{\rm L}} \simeq \epsilon_{u_{\rm L}}^\dagger V_{u_{\rm L}} V, && (5.86)
\end{aligned}
$$

where the approximate unitarity $V_{q_{\rm L}} V_{q_{\rm L}}^\dagger \simeq \mathbf{1}$ is considered up to the second order of the small q-Q mixing. By applying Eq. (5.51) for the Z-boson coupling to Eqs. (5.85) and (5.86), we obtain Eqs. (5.52) and (5.53).

The left-handed Z-boson coupling is given originally in the electroweak basis (q_0, Q_0) as

$$
\mathcal{Z}_{\mathcal{Q}}^{(0)} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} - a \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}, \tag{5.87}
$$

where $a = \sin^2\theta_W Q_{\rm em}(\mathcal{Q})/I_3(q_0)$, and the division by $I_3(q_0) = \pm 1/2$ is for convenience of notation. It is transformed in the basis of mass eigenstates (q, Q) as

$$
\begin{aligned}
\mathcal{Z}_{\mathcal{Q}} &= \mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \mathcal{Z}_{\mathcal{Q}}^{(0)} \mathcal{V}_{\mathcal{Q}_{\rm L}} \\
&= \mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} - a \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}. && (5.88)
\end{aligned}
$$

The modification of the Z-boson coupling due to the q-Q mixing is calculated from Eqs. (5.87) and (5.88) as

$$
\begin{aligned}
\Delta\mathcal{Z}_{\mathcal{Q}} &= \mathcal{Z}_{\mathcal{Q}} - \mathcal{Z}_{\mathcal{Q}}^{(0)} \\
&= \mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} - \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \\
&= \begin{pmatrix} -\epsilon'_{q_{\rm L}} \epsilon'^\dagger_{q_{\rm L}} & V_{q_{\rm L}}^\dagger \epsilon_{q_{\rm L}} \\ \epsilon_{q_{\rm L}}^\dagger V_{q_{\rm L}} & \epsilon_{q_{\rm L}}^\dagger \epsilon_{q_{\rm L}} \end{pmatrix}. && (5.89)
\end{aligned}
$$

Here, we have considered the relation $V_{q_{\rm L}}^\dagger V_{q_{\rm L}} - \mathbf{1} = -\epsilon'_{q_{\rm L}} \epsilon'^\dagger_{q_{\rm L}}$ from the unitarity of $\mathcal{V}_{\mathcal{Q}_{\rm L}}$ to obtain Eq. (5.22) for the upper diagonal block $\Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger q]$ in $\Delta\mathcal{Z}_{\mathcal{Q}}$. We also obtain the Z boson q-Q couplings in Eq. (5.51) from the off-diagonal blocks in Eqs. (5.88) and (5.89) as

$$
\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q] = \Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q] = V_{q_{\rm L}}^\dagger \epsilon_{q_{\rm L}} = (\mathcal{Z}_{\mathcal{Q}}[Q^\dagger q])^\dagger. \tag{5.90}
$$

We note the relation from Eq. (5.89) as

$$
\mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} = \Delta\mathcal{Z}_{\mathcal{Q}} + \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}. \tag{5.91}
$$

By taking the difference between this relation and that for the unit matrix

$$\mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}, \tag{5.92}$$

we obtain another relation

$$\mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} = -\Delta\mathcal{Z}_{\mathcal{Q}} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}. \tag{5.93}$$

By using Eq. (5.91), we calculate

$$\begin{aligned} \mathcal{V}_{\mathcal{Q}_{\rm R}}^\dagger \mathcal{M}_{\mathcal{Q}} \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} &= \bar{\mathcal{M}}_{\mathcal{Q}} \mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} \\ &= \bar{\mathcal{M}}_{\mathcal{Q}} \Delta\mathcal{Z}_{\mathcal{Q}} + \begin{pmatrix} \bar{M}_q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}. \end{aligned} \tag{5.94}$$

On the other hand, by considering Eq. (5.6) for M_q and Δ'_{qQ} in $\mathcal{M}_{\mathcal{Q}}$ we obtain $\Lambda_{\mathcal{Q}}^H$ in Eq. (5.26) as

$$\mathcal{V}_{\mathcal{Q}_{\rm R}}^\dagger \mathcal{M}_{\mathcal{Q}} \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} = \frac{v}{\sqrt{2}} \mathcal{V}_{\mathcal{Q}_{\rm R}}^\dagger \begin{pmatrix} \lambda_q & \mathbf{0} \\ h_q & \mathbf{0} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}}. \tag{5.95}$$

Comparison of Eqs. (5.94) and (5.95) establishes the relation between the Z-boson coupling and the standard Higgs H coupling,

$$\Lambda_{\mathcal{Q}}^H = (\bar{\mathcal{M}}_{\mathcal{Q}}/v)\Delta\mathcal{Z}_{\mathcal{Q}} + \begin{pmatrix} \bar{M}_q/v & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}. \tag{5.96}$$

Specifically, Eq. (5.31) is obtained from the upper diagonal block, and Eqs. (5.54) and (5.55) from the off-diagonal blocks with $\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q] = \Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q]$.

Similarly, we obtain the relation between the Z-boson coupling and the singlet Higgs S_+ coupling as follows. By using Eq. (5.93), we calculate

$$\begin{aligned} \mathcal{V}_{\mathcal{Q}_{\rm R}}^\dagger \mathcal{M}_{\mathcal{Q}} \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} &= \bar{\mathcal{M}}_{\mathcal{Q}} \mathcal{V}_{\mathcal{Q}_{\rm L}}^\dagger \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} \\ &= -\bar{\mathcal{M}}_{\mathcal{Q}} \Delta\mathcal{Z}_{\mathcal{Q}} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{M}_Q \end{pmatrix}. \end{aligned} \tag{5.97}$$

On the other hand, by considering Eq. (5.7) for Δ_{qQ} and M_Q in $\mathcal{M}_{\mathcal{Q}}$ we obtain $\Lambda_{\mathcal{Q}}^{S_+}$ in Eq. (5.27) as

$$\mathcal{V}_{\mathcal{Q}_{\rm R}}^\dagger \mathcal{M}_{\mathcal{Q}} \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}} = \frac{v_S}{\sqrt{2}} \mathcal{V}_{\mathcal{Q}_{\rm R}}^\dagger \begin{pmatrix} \mathbf{0} & f_Q^+ \\ \mathbf{0} & \lambda_Q^+ \end{pmatrix} \mathcal{V}_{\mathcal{Q}_{\rm L}}. \tag{5.98}$$

Comparison of Eqs. (5.97) and (5.98) leads to the expected relation

$$\Lambda_{\mathcal{Q}}^{S_+} = -(\bar{\mathcal{M}}_{\mathcal{Q}}/v_S)\Delta\mathcal{Z}_{\mathcal{Q}} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{M}_Q/v_S \end{pmatrix}. \tag{5.99}$$

Specifically, Eq. (5.32) is obtained from the upper diagonal block, and Eqs. (5.56) and (5.57) from the off-diagonal blocks with $\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q] = \Delta\mathcal{Z}_{\mathcal{Q}}[q^\dagger Q]$.

Chapter 6

Summary and conclusions

The existence of singlet quarks, D with $Q_{\rm em} = -1/3$ and U with $Q_{\rm em} = 2/3$, is one of very intriguing candidates for new physics beyond the standard model of elementary particles, as suggested by the $\rm E_6$-type unified theory. In this thesis, we have investigated extensively the electroweak interaction processes involving the singlet quarks.

We have first presented the systematic and comprehensive investigations on the quark mixings in the electroweak models with singlet quarks. We have examined in detail how the ordinary quark masses and mixings are affected by the q-Q mixing. The flavor changing interactions are also modified by the q-Q mixing. Specifically, the CKM unitarity within the ordinary quark sector is violated, and the FCNC's arise both in the gauge and scalar couplings. The structures of these flavor changing interactions have been clarified in the suitable quark bases. In fact, they are described appropriately in terms of the q-Q mixing parameters and the quark masses. These results ensure that there are some reasonable ranges of the model parameters where the ordinary quark mass hierarchy and the actual CKM structure are reproduced even in the presence of singlet quarks. A detailed numerical analysis has further been performed for calculating precisely the quark mixings and flavor changing couplings with singlet quarks. Then, it has been confirmed that the q-Q mixing effects really exhibit the expected flavor structures. These calculations on the singlet quarks may be extended readily for the models with various exotic quarks and leptons such as vector-like fermions.

We have then investigated the quark masses and mixings specifically in the USY scheme by including down-type quark singlets. In contrast with the standard model with USY, the sufficient CP violation is obtained for the CKM matrix through the mixing between the ordinary down-type quarks and quark singlets. Two or more quark singlets are needed to have the relevant large USY phases for the desired CP violation. These quark singlets may have masses $\sim$ TeV, to be discovered in the future collider experiments, especially at the LHC. We have shown that with rather flexible choices of the USY phase values the actual quark masses and CKM matrix are really reproduced. Then, it is interesting for further investigations to invoke some textures and flavor symmetries for the USY phases so as to derive some predictive relations among the quark masses and mixings. The top-bottom hierarchy $m_t \gg m_b$ also appears naturally in the USY scheme in the presence of extra down-type quark singlets but no extra up-type quark singlets. Furthermore, in the USY scheme (or more generally flavor democracy), the fermion mass hierarchy may be extended as $m_t \gg m_b \sim m_\tau$ if vector-like lepton doublets are also present.

Finally, we have investigated the flavor changing interactions in the model with singlet quarks and singlet Higgs, which are induced by the q-Q mixing. While the gauge interactions have been investigated extensively in the literature, we have rather noted here that the scalar interactions mediated by the singlet Higg may provide significant effects in some cases. This possibility has not been paid so much attention before in the models with singlet quarks. We have considered the effects of the gauge and scalar interactions in the $\Delta F = 2$ mixings of the neutral mesons to show the currently allowed range of the q-Q mixing. Then, we have investigated the decays of the singlet quarks and Higgs particles as the new physics around the electroweak scale to the TeV scale, which is accessible to the LHC. Especially, the right-handed q-Q mixing may be tolerably large without contradicting the current bounds on the flavor changing processes, since it is not involved directly in the electroweak gauge interactions. If this is the case, the scalar coupling by the singlet Higgs, and possibly through the Higgs mixing involving the standard Higgs, provides distinct signatures for the decays of the singlet quarks and Higgs particles, which should be compared with the conventionally expected ones via the gauge and standard Higgs couplings. We expect that observations of the singlet quarks and Higgs particles will present us important insights on the q-Q mixing and Higgs mixing.

These results on the effects of the singlet quarks in the electroweak interaction, including the quark mixing and flavor changing processes, really provide promising perspectives for the new physics in the TeV region beyond the standard model. The discovery of the singlet quarks will provide an important challenge for the experiments at the LHC, starting recently.

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